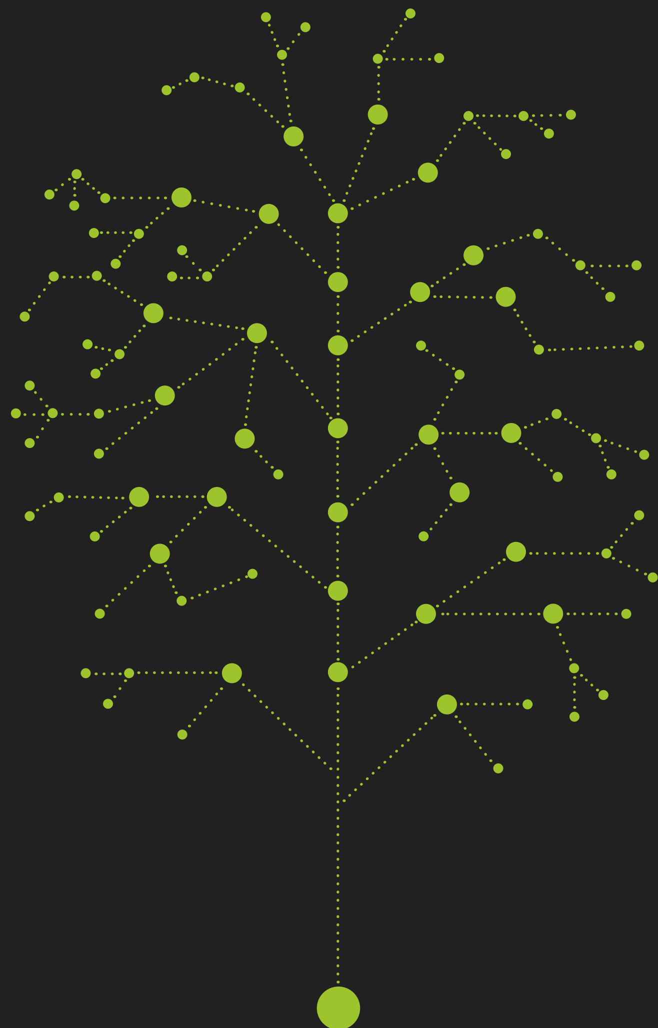

environmental engineering

Applied Hydraulics.
Introduction and basics

Tomasz Siuta

Cracow University of Technology

Kraków 2020

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Table of Contents

Programme description

The purpose of the course is the acquisition of theoretical and practical knowledge in the field of hydraulics of open channel flow in the context of hydraulic structure design. Emphasis will be especially placed upon mastering the ability to perform analytical calculations, as well as on the ability to use the computer program (Hec-Ras) to simulate natural flow scenarios and changes in these conditions due to water structure construction within cross sections of the riverbed.

Programme description

Specific targets of the course:

- the acquisition of theoretical and practical knowledge in the field of calculation of flow parameters in open channels,
- study of the transient flow based upon examples of hydraulic jump and depression curve effect,
- learning of calculation methods for the hydraulic design of: spillways and gated weirs, road culverts, bridges, as well as stilling basins and energy dissipation devices,
- acquiring the ability to carry out computer simulation and analysis of the flow conditions prevailing above and below hydraulic structures, such as spillways and road culverts.

Course types

Lectures (15 h):

1. Basics of open channel flow hydraulics.
2. Non uniform flow analysis.
3. Energy characteristics of the open channel flow.
4. Hydraulic coupling.
5. Weir characteristics and design.
6. Bridge and culverts.

Course types

Practical classes (20 h):

1. Calculation (based on diagrams and formulas) of weir parameters (size of opening, coefficients and etc.) and overflow characteristics for given design and exploitation flow discharges.
2. Simulation of open channel flow in natural and built-up (by hydraulic structure) conditions based on Hec-Ras software – assignment 1 (student report).
3. Numerical simulation of optimal regulation of controlled spillway discharges on the example of a cascade of small objects that perform the functions of a small power plant, and analysis of its effectiveness – assignment 2 (student report).

Course types

4. Calculation of dimensions and parameters of road culverts based on the design flow discharge and simulation of their impact on flow conditions by using the Hec Ras program.
5. Hydraulic design of a stilling basin – calculations based on the mathematical formulas and diagrams and simulation of transient flow based on utilization of the Hec-Ras program.

Course types

Laboratory classes (15 h):

1. Experimental determination of the rating curve for a sharp crested weir.
2. Experimental determination of the ogee shaped weir characteristics: pressure profile-flow rate depended, observation of velocity currents and turbulent flow features.
3. Sluice gate outflow – measurement of conjugated depths and hydraulic jump space and flow characteristics.

Course types

4. Pipe flow – experimental determination of friction and local energy head loss (comparison with theoretical loss coefficient value magnitudes).
5. Water hammer effect in pipe – pressure wave experimental determination and comparison with theoretical calculation of celerity and pressure amplitude.

Requirements for course completion credit

- attendance in laboratory classes (no unjustified absence is demanded), project classes (one unjustified absence is allowed) and lectures (three unjustified absence are allowed),
- oral examination concerning assignments (reports) realized in the frame of practical classes,
- oral examination concerning assignments (reports) realized in the frame of laboratory classes.

Compilation of component grades:

The module grade in semester I = (laboratory classes grade *0.5) + (project classes grade * 0.5)

Part 1

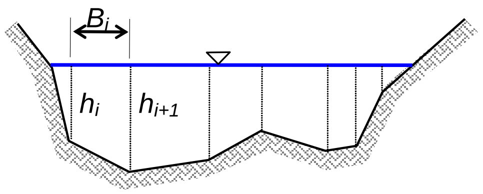
Open channel flow basics

Topics

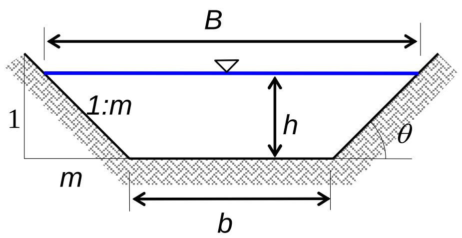
- uniform flow – Manning formula,
- water profile calculation method,
- flow rate calculation within compound channels.

Open channel

Cross-section of the channel:



the natural channel



the man-made channel

size of trapezium shape:

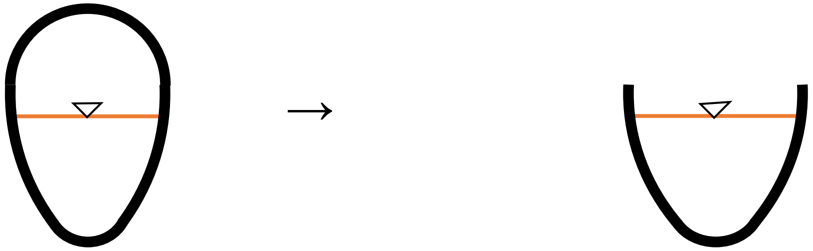
b – width at the bottom [m],

h – maximum depth [m],

m – slope of embankment [–]; $m = \text{ctg}\ \theta$

Closed shape of channels:

collector



Recomended side slopes for channels

Suitate side slopes for channels built in various types of materials (Chow, 1959)

Material	Side slope
Rock	Nearly vertical
Muck and peat soils	1/4 : 1
Stiff clay or earth with concrete lining	1/2 : 1 to 1 : 1
Earth with stone lining or each for large channes	1 : 1
Firm clay or earth for small ditches	1 1/2 : 1
Loose, sandy earth	2 : 1
Sandy loam or porous clay	3 : 1

Geometry of the channels

Prismatic channels:

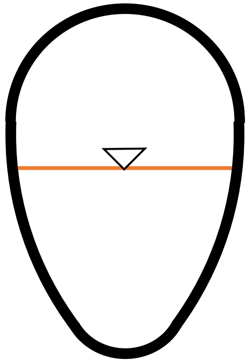
The shape is constant along the channel: $\mathrm{d}A/\mathrm{d}x = 0 \longrightarrow A(h)$,
area of cross-section depends only on depth

Types:

- compact correct: without discontinuity of the shape line
- complex shapes: without discontinuity of the shape line; for example, natural river cross-sections with floodplains

Cylindrical cross-sections:

applied in sewage systems
e.g. circular and egg collectors



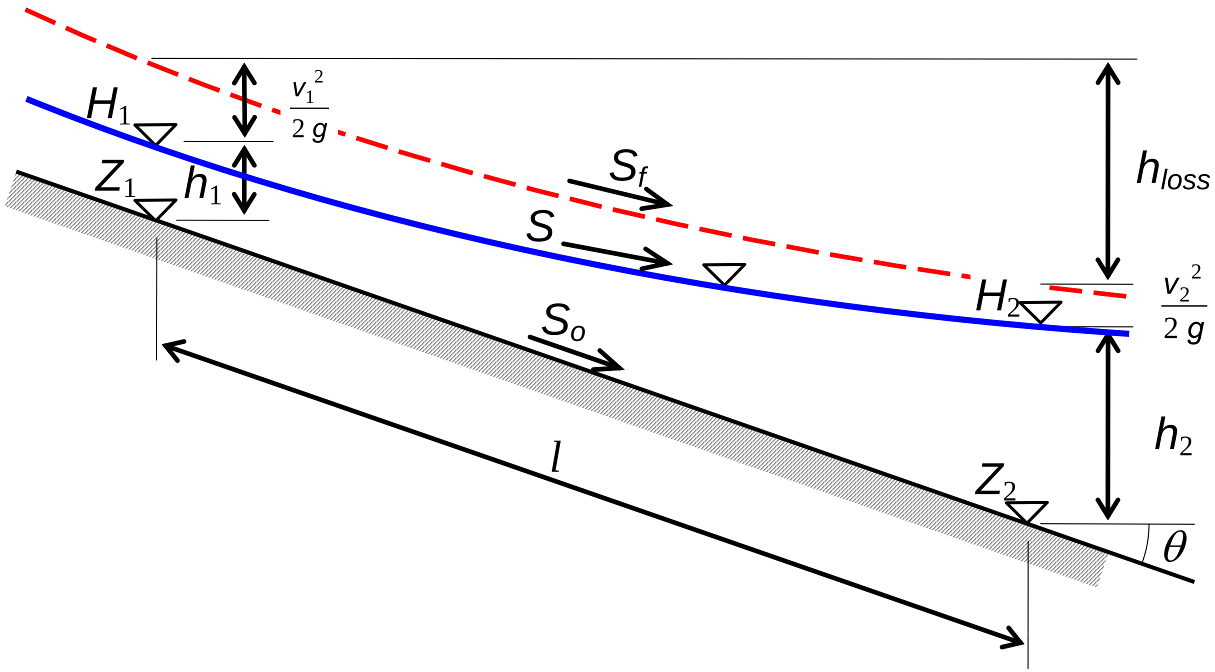
Irregular channels:

The shape varies along the channel: $\mathrm{d}A/\mathrm{d}x \neq 0 \longrightarrow A(h,x)$

Longitudinal profile of a water surface

Bernoullie equation:

$$z_1 + h_1 + \frac{v_1^2}{2g} = z_2 + h_2 + \frac{v_2^2}{2g} + S_f\, l$$



Slope characteristics:

✓ bottom slope S_o:

$$S_o = \frac{z_1 - z_2}{l}$$

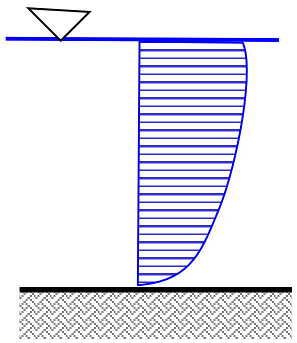
✓ Water-table slope S:

$$S = \frac{H_1 - H_2}{l} = S_o + \frac{h_1 - h_2}{l}$$

✓ grade line slope (energy line) S_f:

$$S_f = \frac{h_{str}}{l} = S + \frac{v_1^2 - v_2^2}{2g}$$

Manning formula



Condition of the open channel flow:

- turbulent flow: Re ≥ 50 000
  second power relation of energy loss – $h_{str} \sim v^2$
- velocity vertical distribution function:
- Saint-Venanta coefficient: α = 1.1

Manning formula:

$$\boxed{v = \frac{\sqrt[3]{R^2}\,\sqrt{S_f}}{n}}$$

where:

R – hydraulic radius [m],

n – Manning’s coefficient [s m$^{-1/3}$]

S_f – hydraulic slope [-]

Manning formula

Hydraulic radius:

$$R = \frac{A}{U}$$

where:
A – cross-section area [m^2],
U – wetted perimeter [m]

- for wide cross-section:

$$R = \frac{b\,h + m\,h^2}{b + 2\,h\,\sqrt{1+m^2}} \xrightarrow[b \gg h]{} \frac{b\,h}{b} = h$$

- in natural channel:

$$R \cong \underline{h}$$

Manning coefficient values

Manning's n for Channels

Type of Channel and Description	Minimum	Normal	Maximum
Natural streams - minor streams (top width at floodstage < 100 ft)			
1. Main Channels			
a. clean, straight, full stage, no rifts or deep pools	0.025	0.030	0.033
b. same as above, but more stones and weeds	0.030	0.035	0.040
c. clean, winding, some pools and shoals	0.033	0.040	0.045
d. same as above, but some weeds and stones	0.035	0.045	0.050
e. same as above, lower stages, more ineffective slopes and sections	0.040	0.048	0.055
f. same as "d" with more stones	0.045	0.050	0.060
g. sluggish reaches, weedy, deep pools	0.050	0.070	0.080
h. very weedy reaches, deep pools, or floodways with heavy stand of timber and underbrush	0.075	0.100	0.150
2. Mountain streams, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages			
a. bottom: gravels, cobbles, and few boulders	0.030	0.040	0.050
b. bottom: cobbles with large boulders	0.040	0.050	0.070

Source: Chow, 1959

Manning coefficient values

3. Floodplains			
a. Pasture, no brush			
1.short grass	0.025	0.030	0.035
2. high grass	0.030	0.035	0.050
b. Cultivated areas			
1. no crop	0.020	0.030	0.040
2. mature row crops	0.025	0.035	0.045
3. mature field crops	0.030	0.040	0.050
c. Brush			
1. scattered brush, heavy weeds	0.035	0.050	0.070
2. light brush and trees, in winter	0.035	0.050	0.060
3. light brush and trees, in summer	0.040	0.060	0.080
4. medium to dense brush, in winter	0.045	0.070	0.110
5. medium to dense brush, in summer	0.070	0.100	0.160
d. Trees			
1. dense willows, summer, straight	0.110	0.150	0.200
2. cleared land with tree stumps, no sprouts	0.030	0.040	0.050
3. same as above, but with heavy growth of sprouts	0.050	0.060	0.080
4. heavy stand of timber, a few down trees, little undergrowth, flood stage below branches	0.080	0.100	0.120
5. same as 4. with flood stage reaching branches	0.100	0.120	0.160

Source: Chow, 1959

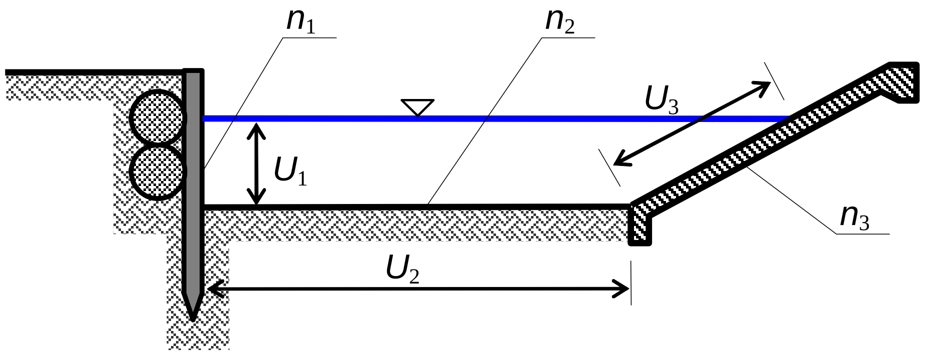
Manning coeficient values

4. Excavated or Dredged Channels			
a. Earth, straight, and uniform			
1. clean, recently completed	0.016	0.018	0.020
2. clean, after weathering	0.018	0.022	0.025
3. gravel, uniform section, clean	0.022	0.025	0.030
4. with short grass, few weeds	0.022	0.027	0.033
b. Earth winding and sluggish			
1. no vegetation	0.023	0.025	0.030
2. grass, some weeds	0.025	0.030	0.033
3. dense weeds or aquatic plants in deep channels	0.030	0.035	0.040
4. earth bottom and rubble sides	0.028	0.030	0.035
5. stony bottom and weedy banks	0.025	0.035	0.040
6. cobble bottom and clean sides	0.030	0.040	0.050
c. Dragline-excavated or dredged			
1. no vegetation	0.025	0.028	0.033
2. light brush on banks	0.035	0.050	0.060
d. Rock cuts			
1. smooth and uniform	0.025	0.035	0.040
2. jagged and irregular	0.035	0.040	0.050
e. Channels not maintained, weeds and brush uncut			
1. dense weeds, high as flow depth	0.050	0.080	0.120
2. clean bottom, brush on sides	0.040	0.050	0.080
3. same as above, highest stage of flow	0.045	0.070	0.110
4. dense brush, high stage	0.080	0.100	0.140

Source: Chow, 1959

Averaging the roughness coefficient

Calculation method:



$$\underline{n} = \frac{\sum U_i\, n_i}{U} = \frac{U_1\, n_1 + U_2\, n_2 + U_3\, n_3}{U_1 + U_2 + U_3}$$

where:

U_i – length of wetted perimeter segment [m] with n_i,

U – total wetted perimeter [m]; U = S U_i

$\underline{n}$ – averaged coefficient

Uniform flow in a channel

Condition of a uniform flow:

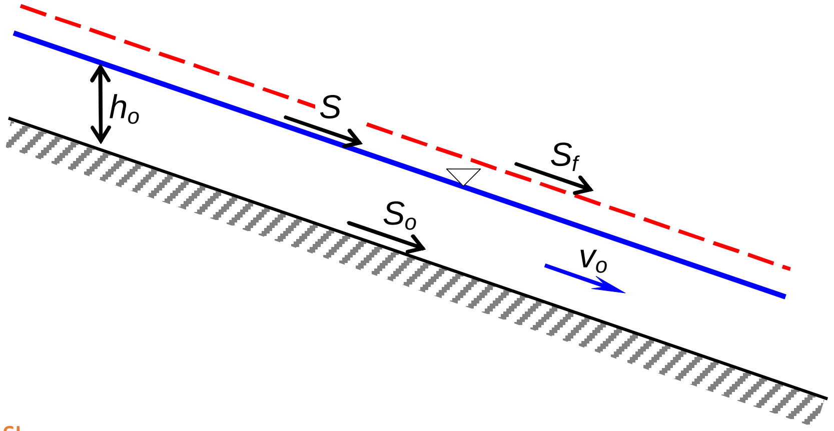
$dv/dx = 0 \longrightarrow v = v_o$ = const – constant velocity value in time and space

v_o – normal velocity component

- prismatic channels: $A(x,h) \longrightarrow A(h)$
- constant depth:

$$Q = A(h)\, v_o = \text{const} \longrightarrow A(h) = \text{const} \longrightarrow h = h_o = \text{const}$$

h_o – normal depth



Slopes in the case of a uniform flow:

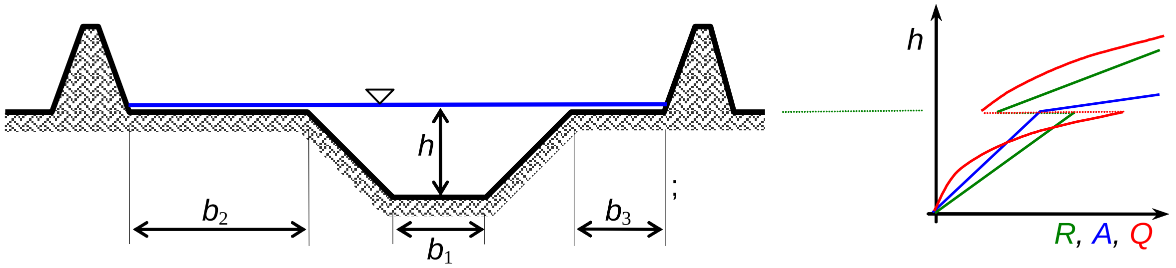
$$S_f = S = S_o$$

Flow rate calculation (Manning formula) in the case of a uniform flow:

$$\boxed{Q = v\,A = A\,\frac{\sqrt[3]{R^2}\,\sqrt{S_o}}{n}}$$

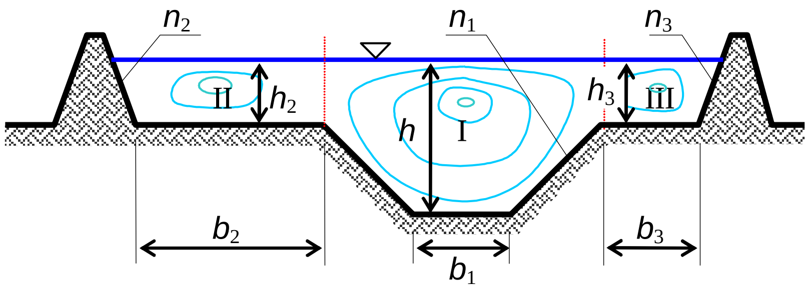
Compound channels

Relation of hydraulic radius $R(h)$:



Division of a compound channel:

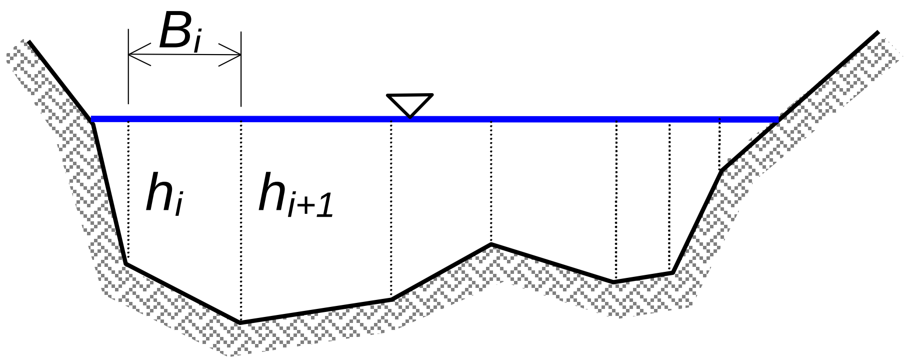
- independent streams,
- vertical division,
- at point of discontinuity.



Flow rate calculation method:

$$\boxed{Q = Q_{\mathrm{I}} + Q_{\mathrm{II}} + \ldots + Q_N = \sum_{i=1}^{N} Q_i = \sqrt{S_o}\sum_{i=1}^{N} A_i \frac{\sqrt[3]{R_i^{\,2}}}{n_i}}$$

Natural river channels



Irregular geometry of cross-section:

vertical zone division of the width B_i

and depths: h_i i h_{i+1}

Compound channels:

division according to the slope magnitude (S)

in the case of S > 1/6-discontnuity

Types of problems to calculate

Variables of flow:

- motion variables: Q, v_o
- depth of water: h_o
- slope of the bottom channel: S_o
- characteristics of cross-sections and hydraulic resistance parameters:

Manning coefficient: n, functions of the depth: $A(h_o)$, $U(h_o)$, $R(h_o)$

Flow rate calculation Q:

given: h, S_o search for: Q i v_o, solution: by substitution

Slope calculation S_o:

given: h, Q, search for : S_o,

solution:

$$S_o = \frac{Q^2}{\left(\sum A_i \frac{\sqrt[3]{R_i^{\,2}}}{n_i}\right)^2}$$

Calculation of the depth

Given: Q, S_o

Search for: h_o

Solution:

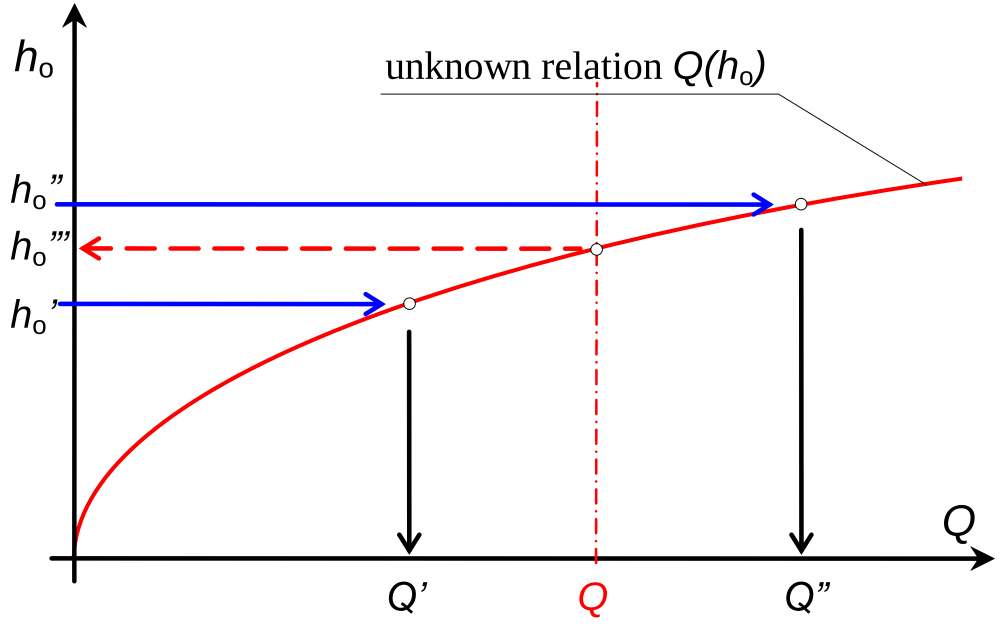
- geometry variables (A, U, R) are the functions of h_o
- iterative calculation for guess value of unknown h_o
- results in the table:

No.	h_o	U	A	R	$\underline{n}$	V	Q
1	1 m						
2							

- comparison of calculated flow rate with given value, new approximation of h_o: $Q \sim h_o$
- convergence criteria: alternate h approximation
- values differ less than 1%

Calculation of the depth

Helpful drawing:



Part 2

Spatially non-uniform flow

Topics

- steady flow vs. unsteady,
- types of non-uniform flow,
- governing equations and tail water profile.

Non-uniform flow vs. unsteady

Unsteady flow:

$$v(x,\, t),\ \frac{\partial v}{\partial x} \neq 0$$

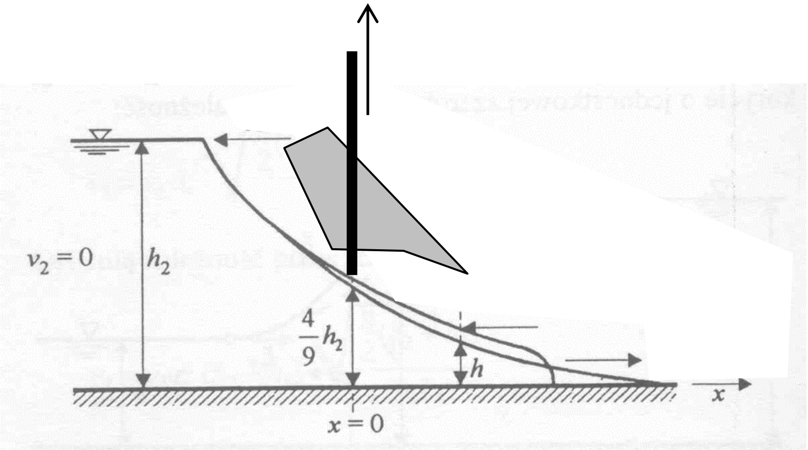
Steady flow:

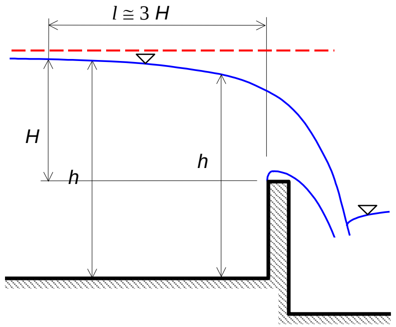
- Non-uniform flow:

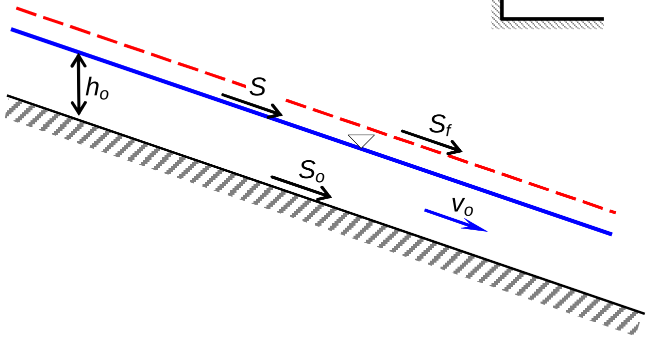
$$v(x),\ \frac{\mathrm{d}v}{\mathrm{d}x} \neq 0$$

- Uniform flow:

$$v = v_o = \text{const},\ \frac{\mathrm{d}v}{\mathrm{d}x} = 0$$



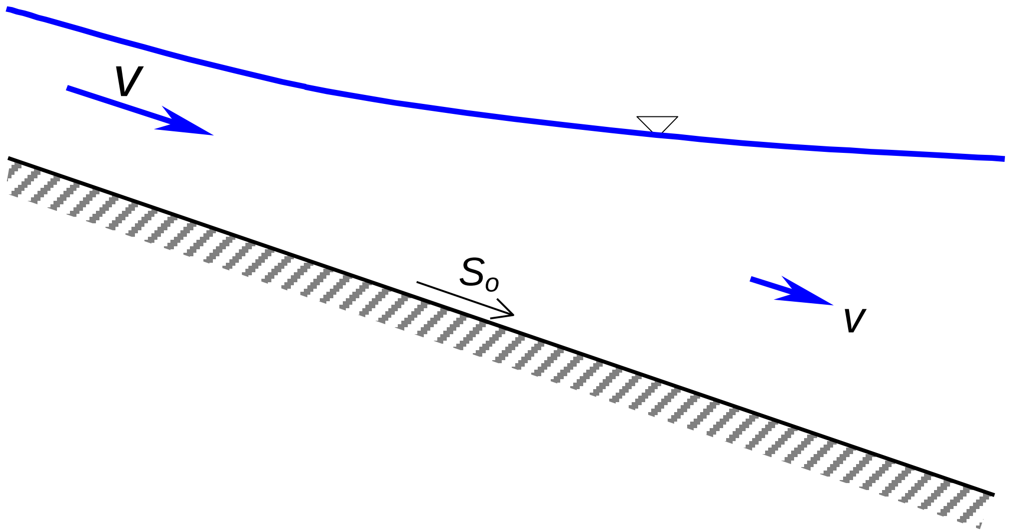




Non-uniform steady flow

Definition:

Flow parameters (v, h, p, Q) are constant in time but vary along the streamline

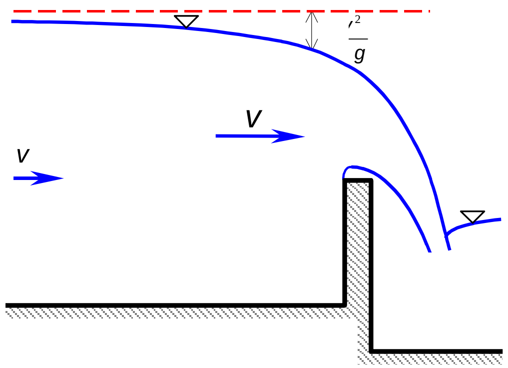


Types of non-uniform flow

Accelerated flow:

Velocity magnitude increases towards flow direction;

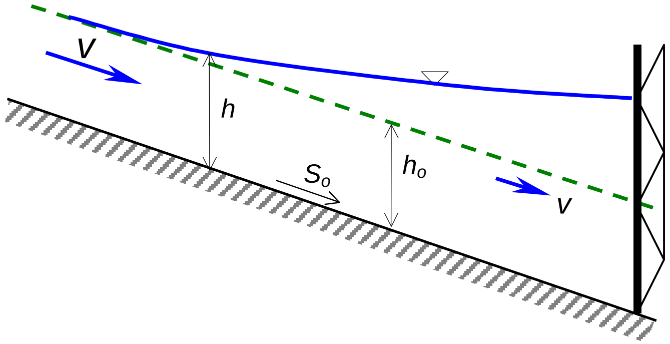
$$\frac{\mathrm{d}v}{\mathrm{d}x} > 0 \;\longrightarrow \text{depression curve}$$



Decelerated flow:

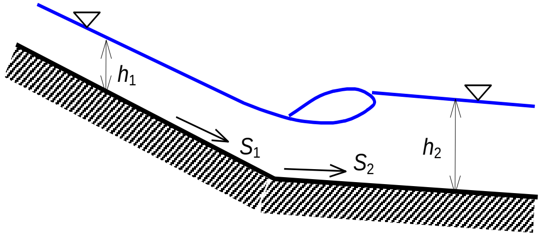
Velocity magnitude decreases towards flow direction;

$$\frac{\mathrm{d}v}{\mathrm{d}x} < 0 \;\longrightarrow \text{damming (tail water)}$$

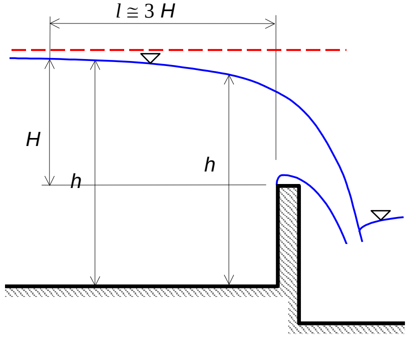


Range of depth changes for different flow conditions

Supercritical flow:
There is no impact of tail water on water-table profile,



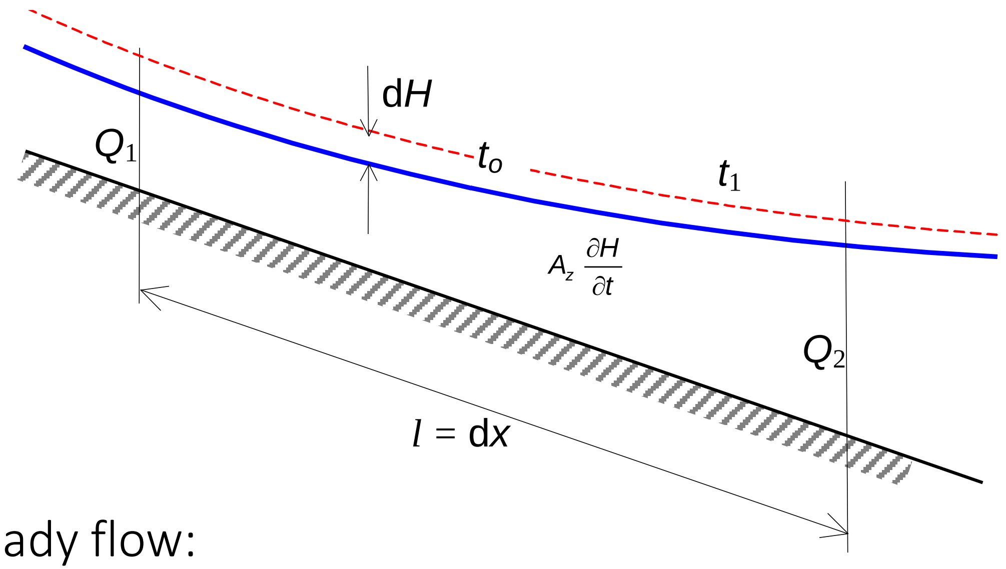
Depression:
Range ≅ (2÷3) H



Back water:
In same cases, the range may be a couple of hundred km (e.g. The Cymlański reservoir of the Wołga river)

Continuity equation

$$Q_2 = Q_1 + Q_b - A_z \frac{\partial H}{\partial t} \longrightarrow \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_b$$



In the case of steady flow:

$$\frac{dQ}{dx} = 0 \longrightarrow \frac{dQ}{dx} = \frac{dv\,A}{dx} = v\frac{dA}{dx} + A\frac{dv}{dx} = 0$$

Basic differential equation for steady non-uniform flow

Saint-Venanta equation

Bernoullie equation:

$$z + h + \frac{\alpha v^2}{2g} + h_{str} = \mathrm{const}$$

Differential Bernoullie equation:

$$\frac{\mathrm{d}z}{\mathrm{d}x} + \frac{\mathrm{d}h}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\alpha v^2}{2g}\right) + S_f = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\alpha v^2}{2g}\right) = \frac{\alpha}{2g} 2v \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\alpha}{g} v \frac{\mathrm{d}v}{\mathrm{d}x} \longrightarrow \boxed{\frac{\alpha}{g} v \frac{\mathrm{d}v}{\mathrm{d}x} + \frac{\mathrm{d}h}{\mathrm{d}x} + S_f - S_o = 0}$$

Momentum equation

Other forms of Saint-Venant equation

Substitution:

$$v = \frac{Q}{A}$$

$$\frac{\alpha\, Q}{g\, A}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{\mathrm{d}h}{\mathrm{d}x} + S_f - S_o = 0$$

Conservation form:

$$\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{v}{A}\frac{\mathrm{d}A}{\mathrm{d}x} = -\frac{Q}{A^2}\frac{\mathrm{d}A}{\mathrm{d}x}$$

$$\longrightarrow \quad \boxed{-\frac{\alpha\, Q^2}{g\, A^3}\frac{\mathrm{d}A}{\mathrm{d}x} + \frac{\mathrm{d}h}{\mathrm{d}x} = S_o - S_f}$$

Back water profile equation

General equation:

for $\dfrac{dA}{dx} = \dfrac{dA}{dh}\dfrac{dh}{dx} = B\dfrac{dh}{dx}$ and $S_f = \dfrac{n^2 Q^2}{A^2 R^{4/3}}$

after substitution

$$-\frac{\alpha B Q^2}{g A^3}\frac{dh}{dx} + \frac{dh}{dx} = S_o - \frac{n^2 Q^2}{A^2 R^{4/3}}$$

hence $\boxed{\dfrac{dh}{dx} = \dfrac{S_o - \dfrac{n^2 Q^2}{A^2 R^{4/3}}}{1 - \dfrac{\alpha B Q^2}{g A^3}}}$ or $\boxed{\dfrac{dh}{dx} = S_o \dfrac{1 - \dfrac{n^2 Q^2}{S_o A^2 R^{4/3}}}{1 - \dfrac{\alpha B Q^2}{g A^3}}}$

Back water profile equation for rectangular cross-section channel

Substitution:

$A = b\,h$, a for $B \longrightarrow \infty : R \longrightarrow h$ thus

$$\frac{n^2\,Q^2}{S_o\,A^2\,R^{4/3}} = \frac{n^2\,Q^2}{S_o\,B^2\,h^{3\,1/3}} \cong \frac{h_o{}^3}{h^3}$$

$$\frac{\alpha\,B\,Q^2}{g\,A^3} = \frac{\alpha\,Q^2}{g\,B^2\,h^3} = \frac{h_k{}^3}{h^3}$$

For any slope:

$$\boxed{\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_o - \dfrac{Q^2}{K^2}}{1 - \dfrac{h_k{}^3}{h^3}}}$$

$$h_o{}^3 = \frac{n^2\,Q^2}{S_o\,B^2\,h_o{}^{1/3}}$$

K – discharge convoyence

$$\frac{Q^2}{S_o}\,\frac{n^2}{A^2\,R^{4/3}} = \frac{K_o{}^2}{K^2}$$

For positive slope:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = S_o\,\frac{1 - \dfrac{K_o{}^2}{K^2}}{1 - \dfrac{h_k{}^3}{h^3}} \cong S_o\,\frac{1 - \dfrac{h_o{}^3}{h^3}}{1 - \dfrac{h_k{}^3}{h^3}}$$

$$\boxed{\frac{\mathrm{d}h}{\mathrm{d}x} = S_o\,\frac{h^3 - h_o{}^3}{h^3 - h_k{}^3}}$$

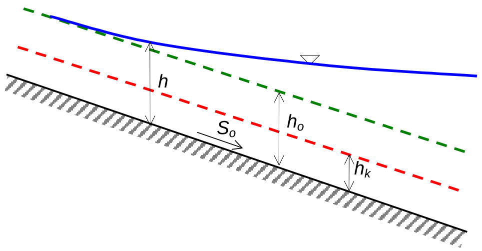
Discussion of the equation profile solution for different flow conditions

Positive slope: $S_o > 0$ hence $\quad \dfrac{dh}{dx} \sim \dfrac{h - h_o}{h - h_k},\qquad dh > 0$ – damming,
$dh < 0$ – depression,

Subcritical flow:

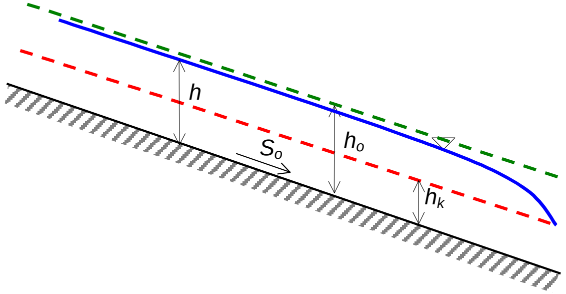
- damming:
  $h > h_o > h_k$

$$\frac{h - h_o}{h - h_k} > 0 \quad \Rightarrow \quad \frac{dh}{dx} > 0$$



- depression:
  $h_o > h > h_k$

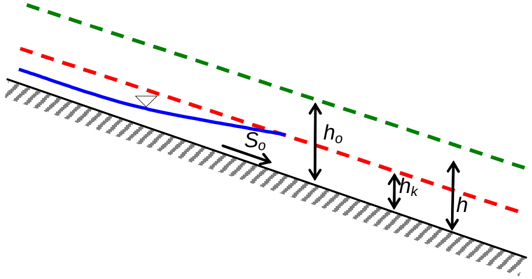
$$\frac{h - h_o}{h - h_k} < 0 \quad \Rightarrow \quad \frac{dh}{dx} < 0$$



Supercritical flow:

- outflow:
  $h_o > h_k > h$

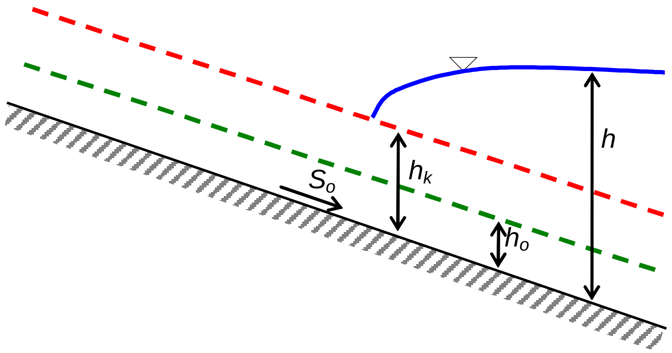
$$\frac{h - h_o}{h - h_k} > 0 \quad \Rightarrow \quad \frac{dh}{dx} > 0$$



Supercritical flow

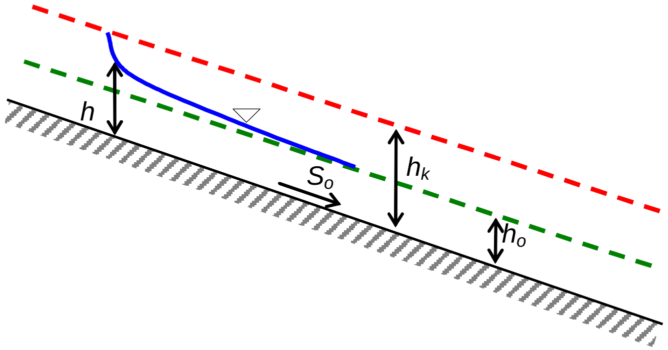
- damming:
  $h > h_k > h_o$

  $$\frac{h - h_o}{h - h_k} > 0 \quad \Rightarrow \quad \frac{dh}{dx} > 0$$



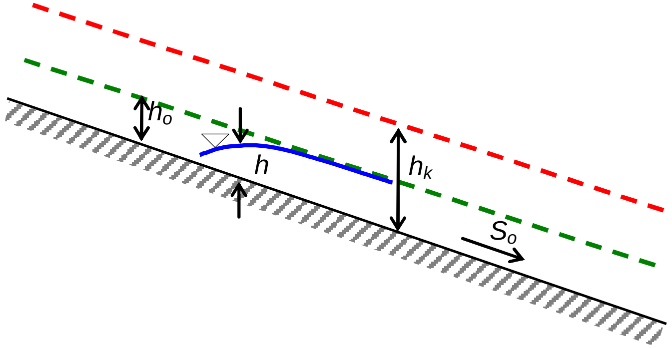
- depression:
  $h_k > h > h_o$

  $$\frac{h - h_o}{h - h_k} < 0 \quad \Rightarrow \quad \frac{dh}{dx} < 0$$



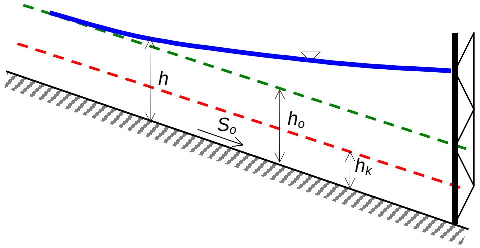
- outflow:
  $h_k > h_o > h$

  $$\frac{h - h_o}{h - h_k} > 0 \quad \Rightarrow \quad \frac{dh}{dx} > 0$$

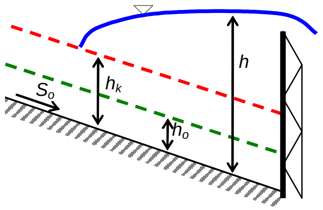


Examples

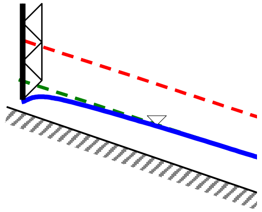
Subcritical damming



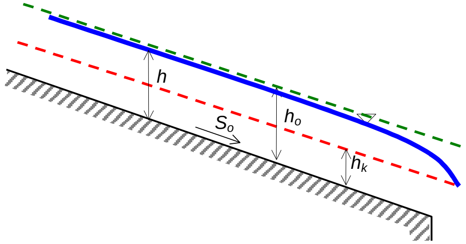
Supercritical damming



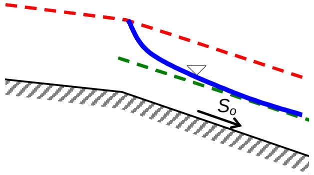
Subcritical outflow



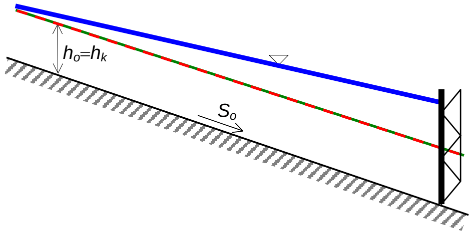
Subcritical depression



Supercritical depression



Critical damming



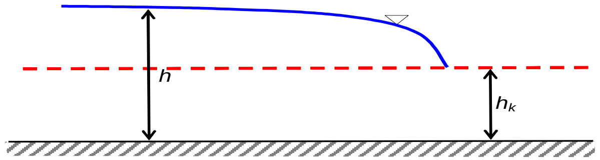
Discussion for zero slope

Zero slope: $S_o = 0$ hence $$\frac{dh}{dx} = \frac{-\frac{Q^2}{K^2}}{1 - \frac{h_k^3}{h^3}} \qquad \frac{dh}{dx} \sim h_k - h$$

Subcritical flow:

$h > h_k$

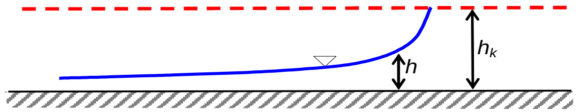
$h_k - h < 0 \quad \Rightarrow \quad \frac{dh}{dx} < 0$



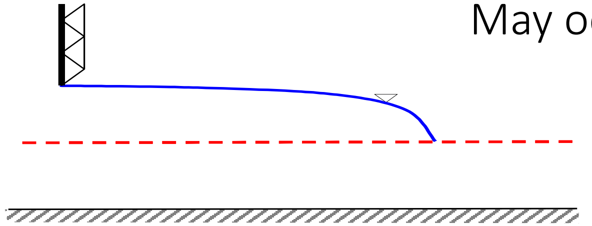
Supercritical flow:

$h < h_k$

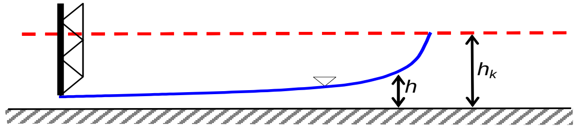
$h_k - h > 0 \quad \Rightarrow \quad \frac{dh}{dx} > 0$



Examples:



May occur in a stilling basin



Discussion for negative slope

Negative slope: $S_o < 0$ hence $\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_o - \frac{Q^2}{K^2}}{1 - \frac{h_k^3}{h^3}}$ $\frac{\mathrm{d}h}{\mathrm{d}x} \sim h_k - h$

Subcritical flow:

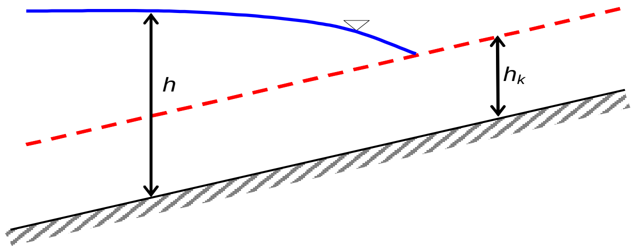
$h > h_k$

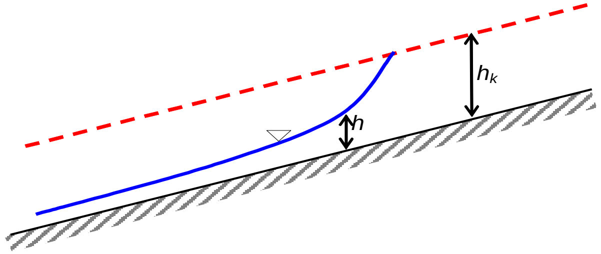
$h_k - h < 0 \quad \Rightarrow \quad \frac{\mathrm{d}h}{\mathrm{d}x} < 0$

Supercritical flow:

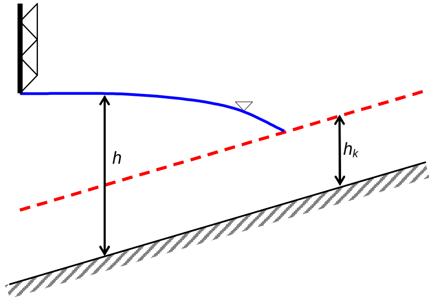
$h < h_k$

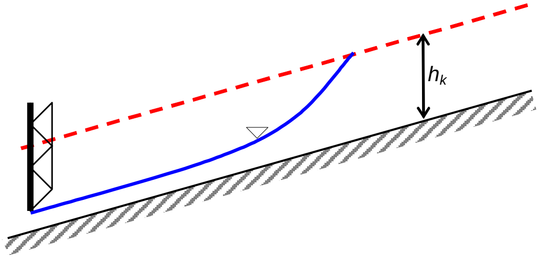
$h_k - h > 0 \quad \Rightarrow \quad \frac{\mathrm{d}h}{\mathrm{d}x} > 0$





Examples:





Part 3

Energy of flow in open channels

Topics

- optimal channel size calculation,
- specific energy as function of water level,
- classification of flow according to Froude number.

The optimal channel size according to the different criteria

The maximum conveyance criteria:

The largest flow rate for a given: area, slope, Manning coefficient

Q_{max} = ?, given A, S_o, n

$Q_{max} \longrightarrow R_{max} \longrightarrow U_{min} \longrightarrow$ semicircle ($r = h$)

Other definition of the optimal channel: the smallest area (volume of ground earth works) for given flow rate, slope, Manning coefficient

A_{min} = ?, given: Q, S_o, n

The smallest slope (the smallest energy loss) for given flow rate, Manning coefficient and cross-section area

S_{min} = ?, given: Q, A, n

The optimal size of the trapezoidal channel cross-section

Optimal depth for given area (A):

$A = b\,h + m\,h^2 \rightarrow b = A/h - m\,h,$ $\qquad M = 2\sqrt{1+m^2} - m$

$$U = b + 2\,h\sqrt{1+m^2} = \frac{A}{h} - m\,h + 2\,h\sqrt{1+m^2} = \frac{A}{h} + M\,h$$

$$\frac{dU}{dh} = -\frac{A}{h^2} - m + 2\sqrt{1+m^2} = -\frac{A}{h^2} + M = 0$$

$\rightarrow A = M\,h^2,\ b = h\,(M - m),\ U = 2\,M\,h,$ $\qquad R = \dfrac{A}{U} = \dfrac{M\,h^2}{2\,M\,h} = \dfrac{h}{2}$

$$\beta_{opt} = \frac{b}{h} = M - m = 2\sqrt{1+m^2} - 2\,m$$

Optimal embankment slope for given depth (h):

$$U = \frac{A}{h} - h\,\mathrm{ctg}\,\theta + 2\,h\sqrt{1+\mathrm{ctg}^2\,\theta} = \frac{A}{h} - h\,\mathrm{ctg}\,\theta + \frac{2\,h}{\sin\theta} = 0$$

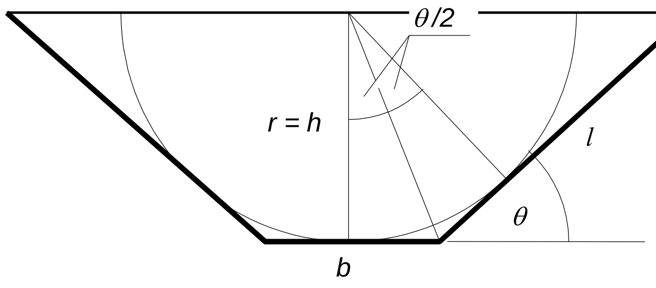
$$\frac{dU}{d\theta} = -h\,(\mathrm{ctg}\,\theta)' + \frac{2\,h}{(\sin\theta)'} = \frac{h}{\sin^2\theta} - \frac{2\cos\theta\,h}{\sin^2\theta} = \frac{h}{\sin^2\theta}(1 - 2\cos\theta) = 0$$

$\rightarrow 1 - 2\cos\theta = 0 \rightarrow \cos\theta = ½ \rightarrow \theta = 60^{o} \rightarrow m = 0.58$

Optimal shape of the channel cross-section

Optimal shape of trapezoidal cross-section:

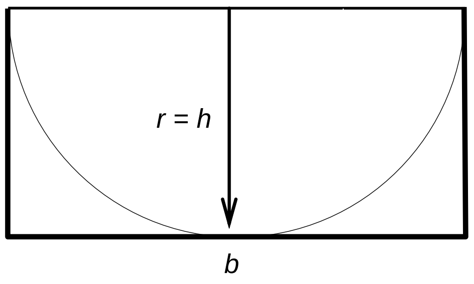
$\theta = 60^o$



$$\beta_{opt} = \frac{b}{h} = 2\sqrt{1+m^2} - 2m$$

$$\beta_{opt} = 2\sqrt{1+0.58^2} - 2\cdot 0.58 = 1.15$$

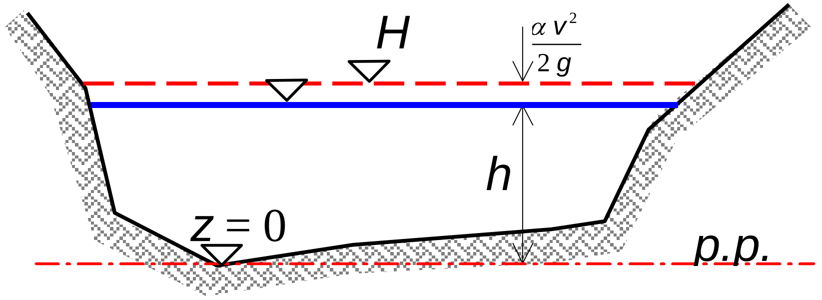
Optimal shape of rectangular cross-section:



$$\beta_{opt} = \frac{b}{h} = 2\sqrt{1+0^2} - 2\cdot 0 = 2$$

Specific energy definition

Total head at the cross-section measured according to the bottom level:



$$H = h + \frac{\alpha v^2}{2g}$$

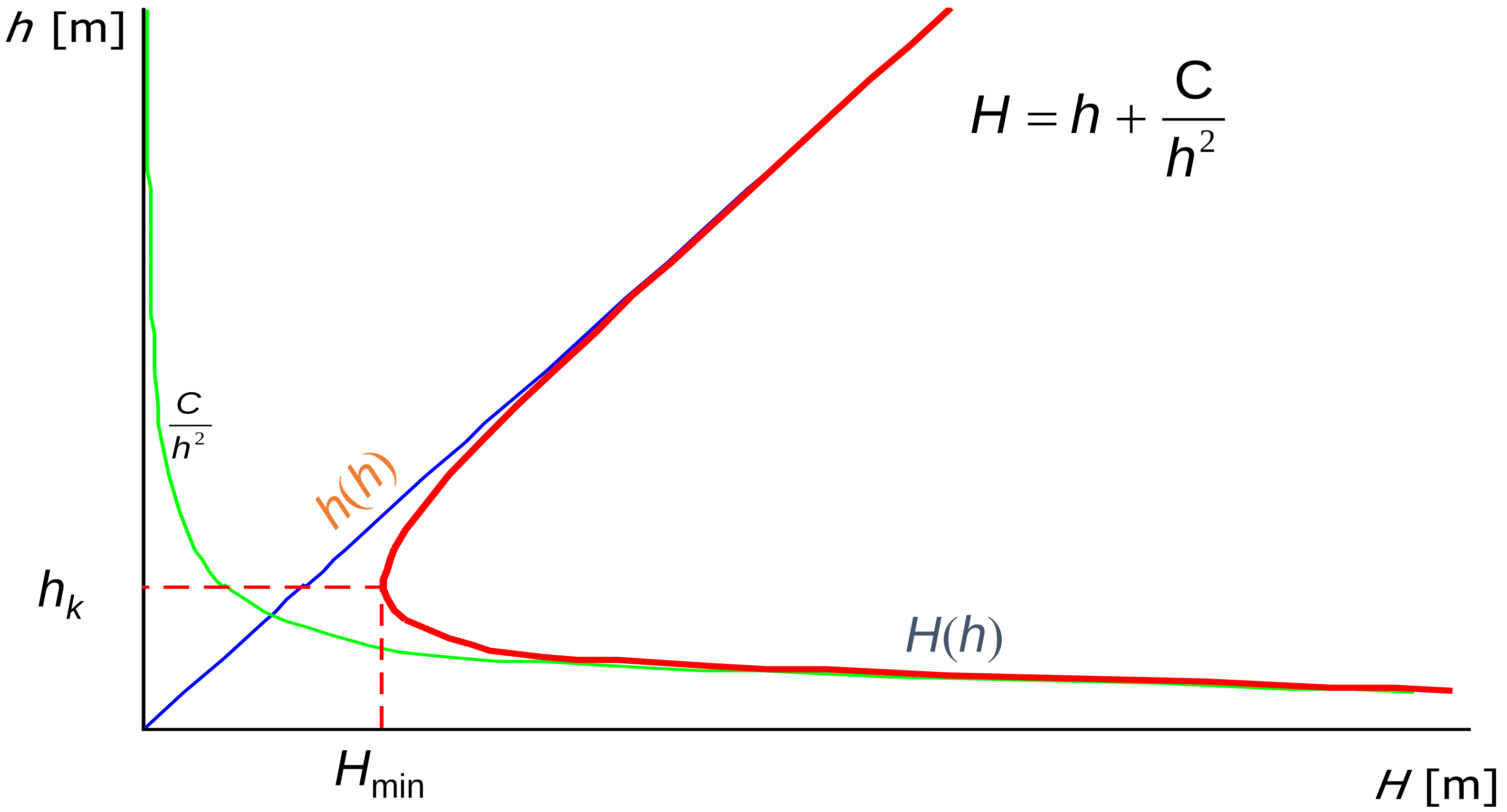
Specific energy as a function of depth for a given flow rate:

$$H = z + h + \frac{\alpha v^2}{2g} = h + \frac{\alpha Q^2}{2g\, b^2\, h^2}$$

for $z = 0$ i

$$v = \frac{Q}{b\, h}$$

The graph of the specific energy function



Critical depth

Definition:

- depth of the flow, at which specific energy for given flow rate is minimal (E_{min}),
- depth of the flow, at which flow rate per unit of the water-table width is maximal for the given value of specific energy (Q_{max}).

Derivation of critical depth in rectangular channel:

$$\min H: \quad \frac{\mathrm{d}H}{\mathrm{d}h} = 1 - \frac{\alpha\, Q^2}{g\, b^2\, h^3} = 0 \qquad \rightarrow \qquad \boxed{h_k = \sqrt[3]{\frac{\alpha\, Q^2}{g\, b^2}}}$$

Energy ratio at critical flow condition:

$$H = h_k + \frac{\alpha\, Q^2}{2\, g\, b^2\, h_k^2} = h_k + \frac{h_k^3}{2\, h_k^2} = h_k + \frac{1}{2} h_k \qquad \rightarrow \qquad \frac{E_v}{E_g} = \frac{1}{2}$$

$$\rightarrow \quad h_k = \frac{2}{3} H \qquad H = \frac{3}{2} h_k = 3\, \frac{\alpha\, v_k^2}{2\, g}$$

Critical velocity

Derivation:

- $v_o(h_o = h_k)$,

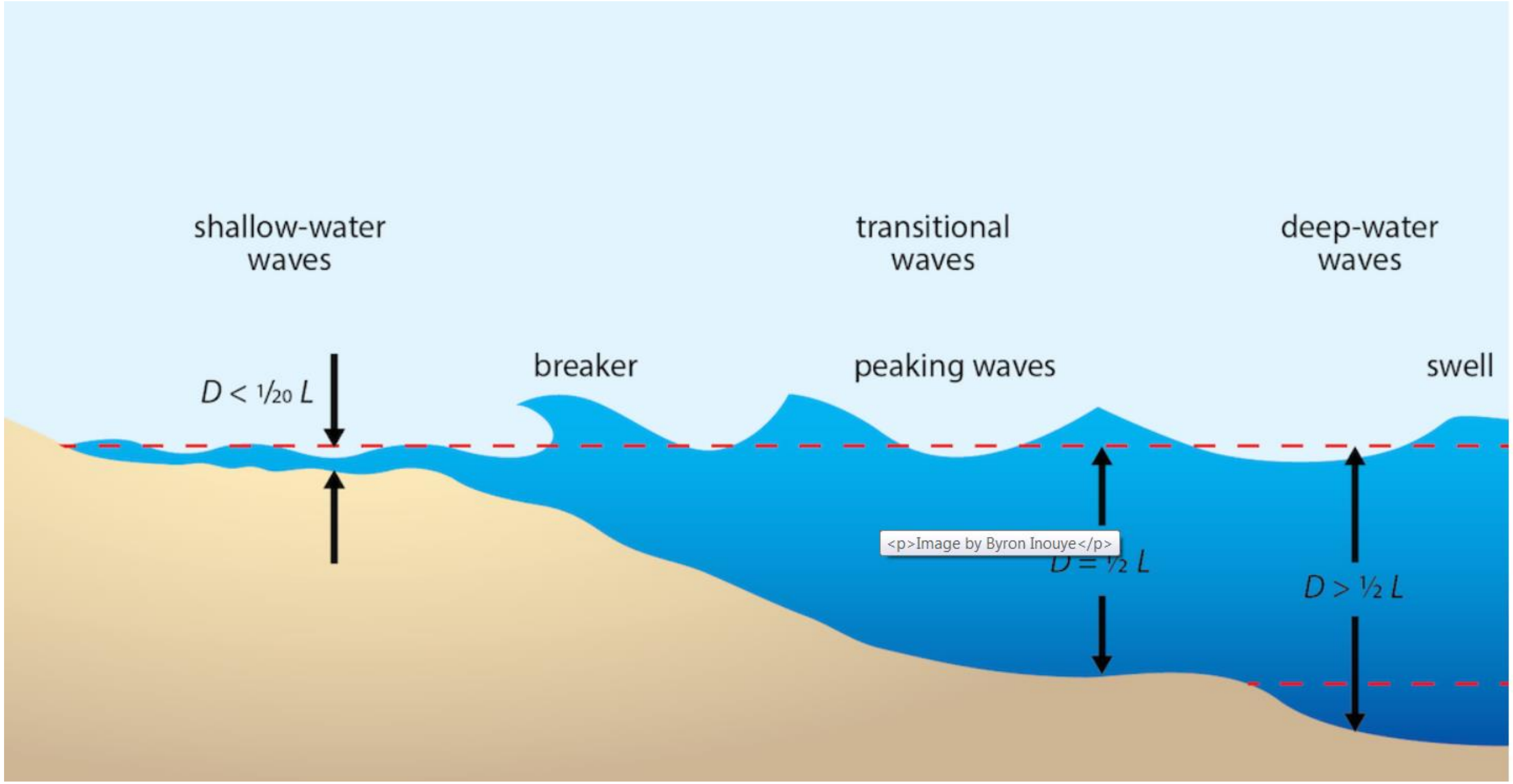
$$h_k^{\,3} = \frac{\alpha\, Q^2}{g\, b^2} = \frac{\alpha\, v^2\, h_k^{\,2}}{g} \qquad \rightarrow \qquad \boxed{v_k = \sqrt{\frac{g\, h}{\alpha}} \cong c}$$

Critical velocity as a celerity of shallow water wave:

$$\boxed{c = \sqrt{g\, h}}$$

h = 1 m: c = 3 m/s = 11 km/h

h = 4 km: c = 200 m/s = 700 km/h

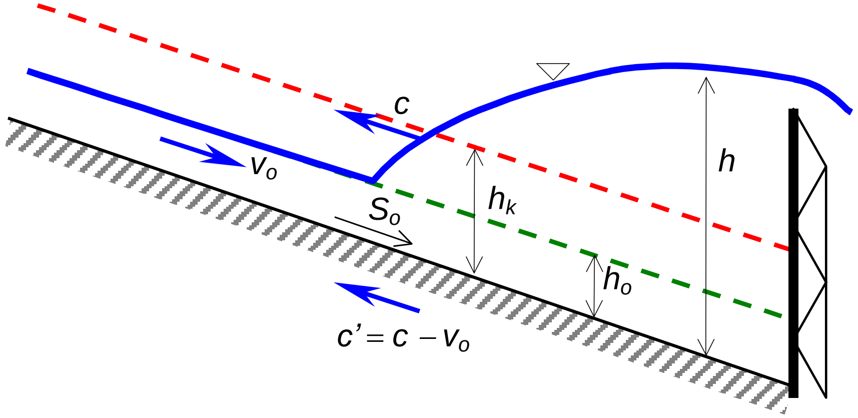


Source: Wave Energy and Wave Changes with Depth

Regime of flow in an open channel

Froude number – Fr:

$$\boxed{\mathrm{Fr} = \frac{v}{\sqrt{\frac{g\,h}{\alpha}}}} = \frac{v}{v_k} = \sqrt{\frac{h_k}{h}} = \sqrt{\frac{2\,E_v}{E_g}}, \quad \mathrm{Fr} = \mathrm{Fr}^2$$



Supercritical flow:

Fr > 1

$c' = c - v_o < 0$

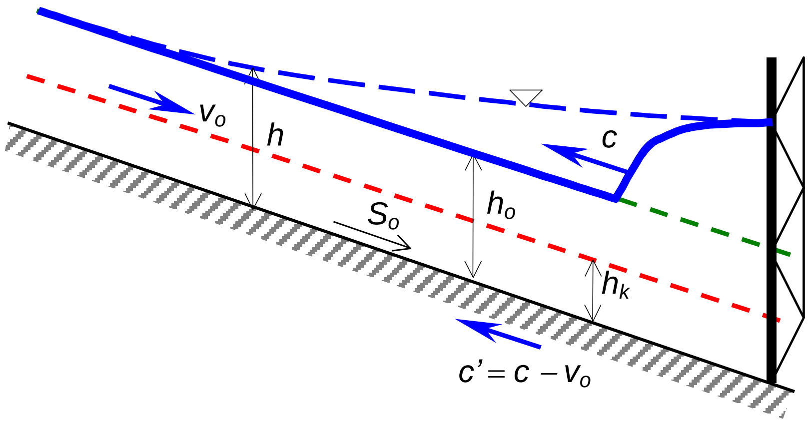
Standing wave

Subcritical flow:

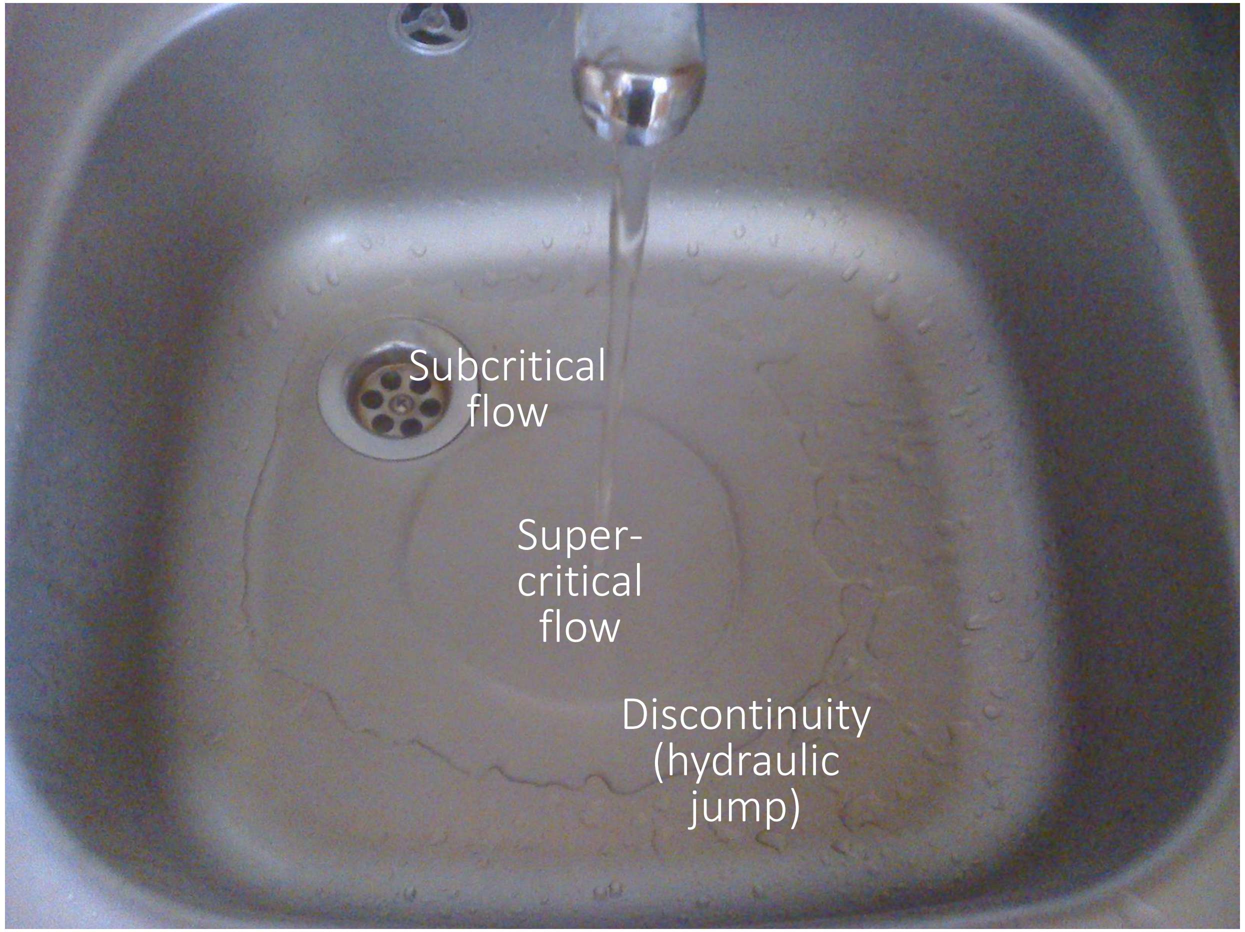
Fr < 1

$c' = c - v_o > 0$

Back water profile



Small scale common hydraulic jump



Critical slope

Derivation and definition:

$S_o(h_o = h_k)$

$$\boxed{S_k = \frac{v_k^2 n^2}{R^{\frac{4}{3}}} = \frac{g\,h\,n^2}{\alpha\,R^{\frac{4}{3}}} \cong \frac{g\,h\,n^2}{\alpha\,h^{\frac{4}{3}}} = \frac{g\,n^2}{\alpha\,h^{\frac{1}{3}}} \cong \text{const}}$$

Mountain rivers or streams:

$S_o > S_k$ – supercritical flow

Sub-mountain rivers:

$S_o \cong S_k$ – regime depends on flow rate

Lowland rivers:

$S_o < S_k$ – generally a subcritical regime of flow

Critical flow criteria for any shape of channel cross-section

Specific energy as a function of depth:

$$H = h + \frac{\alpha Q^2}{2 g A^2}$$

The minimalization of energy criteria:

min H: $$\frac{\mathrm{d}H}{\mathrm{d}h} = 1 - \frac{2 \alpha Q^2}{2 g A^3} \frac{\mathrm{d}A}{\mathrm{d}h} = 0$$ $\rightarrow$ $\mathrm{d}A = B\,\mathrm{d}h$

Condition to be satisfied for critical flow:

$$\boxed{\frac{A^3(h_k)}{B(h_k)} = \frac{\alpha Q^2}{g}}$$

Critical depth (h_k) is derived from this formula based on an iterative numerical solution.

Energy ratio at critical flow condition

Kinetic energy head:

$$\frac{\alpha v^2}{2g} \equiv \frac{\alpha Q^2}{2g A^2(h_k)} = \frac{A(h_k)}{2B(h_k)} \equiv \frac{\underline{h}}{2}$$

Total head:

$$H = h_k + \frac{\alpha v^2}{2g} = h_k + \frac{1}{2} h_k$$

Energy ratio:

- kinetic to potential energy ratio: $\dfrac{E_v}{E_g} = \dfrac{1}{2}$
- calculation of h_k based on energy head: $h_k = \dfrac{2}{3} H$
- calculation of v_k based on energy head: $H = \dfrac{3}{2} h_k = 3 \dfrac{\alpha v_k^{\,2}}{2g}$

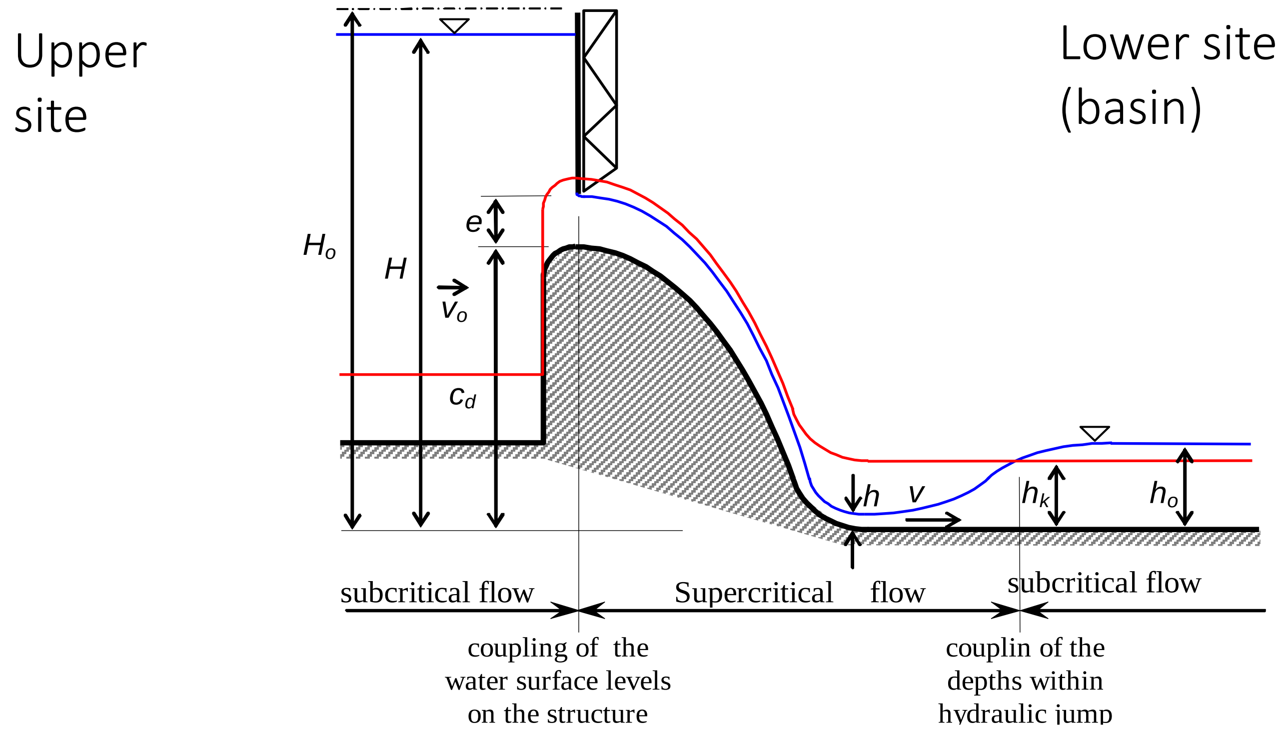
Part 4

Hydraulic coupling

Topics

- transition flow conditions on a spillway,
- hydraulic coupling definition,
- hydraulic jump characteristics and types.

Transition flow caused by damming structure



When water flows through the damming construction, depth changes occur:

- depression curve on the spillway – decreasing from H to h within basin
- return to the normal depth h_o by hydraulic jump phenomena

Hydraulic coupling definition

Depths and speeds in the examined cross-sections are related by energy dependencies.

Hydraulic coupling is for the transition between depths in two cross-sections with different flow regime.

The two critical passages are of two types of hydraulic coupling:

- hydraulic coupling of water-building sites:
  relation $h = \mathrm{f}(H)$
- conjugated depths within hydraulic jump:
  relation between $h \sim h_{critical}$

Application of the Bernoulli equation to derive the coupling depths of the damming structures

Assuming a datum level at the bottom of the basin, total energy heads for upstream and downstream cross-sections are obtained as:

- upstream cross-sections: $H_o = H + \frac{v_o^{\ 2}}{2g}$
- downstream c.s.: $h + \frac{v^2}{2g} + \zeta \frac{v^2}{2g} = h + \frac{1}{\varphi^2} \frac{v^2}{2g}$

the Bernoulli equation looks like:

$$H_o = h + \frac{1}{\varphi^2} \frac{v^2}{2g}$$

Velocity calculation at the downstream cross-section

Solving the equation for the unknown *v*:

$$H_o = h + \frac{1}{\varphi^2}\frac{v^2}{2g} \quad \Longrightarrow \quad v = \varphi\sqrt{2g\left(H_o - h\right)}$$

where:

$\varphi = 0.80 \div 1.00$

φ – is the function of the spillway height and roughness

Calculation of the depth at the downstream cross-section

By adopting a rectangular cross-section for a channel of width *b*, the velocity can be calculated based on the flow rate:

Averaged velocity:

$$v = \frac{Q}{b\,h}$$

hence:

$$\frac{1}{\varphi^2}\frac{v^2}{2g} = \frac{Q^2}{2\,g\,\varphi^2\,b^2\,h^2}$$

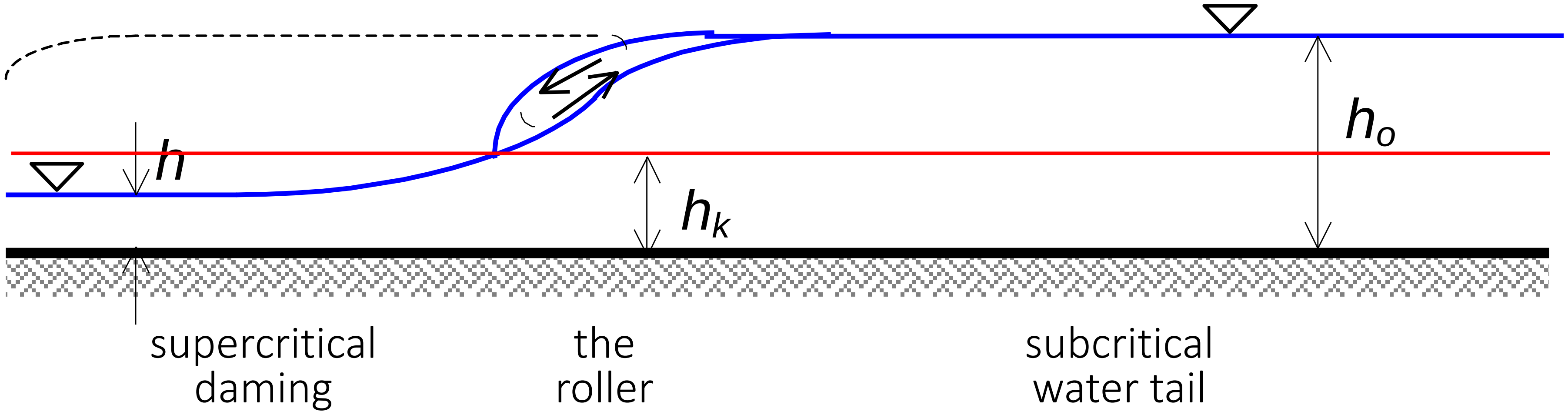
- which when substituted into the Bernoulli equation gives the third degree equation due to *h*:

$$H_o = h + \frac{Q^2}{2\,g\,\varphi^2\,b^2\,h^2}$$

- the searched depth is one of the roots of the equation:

$\cancel{h' < 0}$ $\qquad 0 < h'' < h_k \qquad$ $\cancel{h''' > h_k}$

The Bidone hydraulic jump

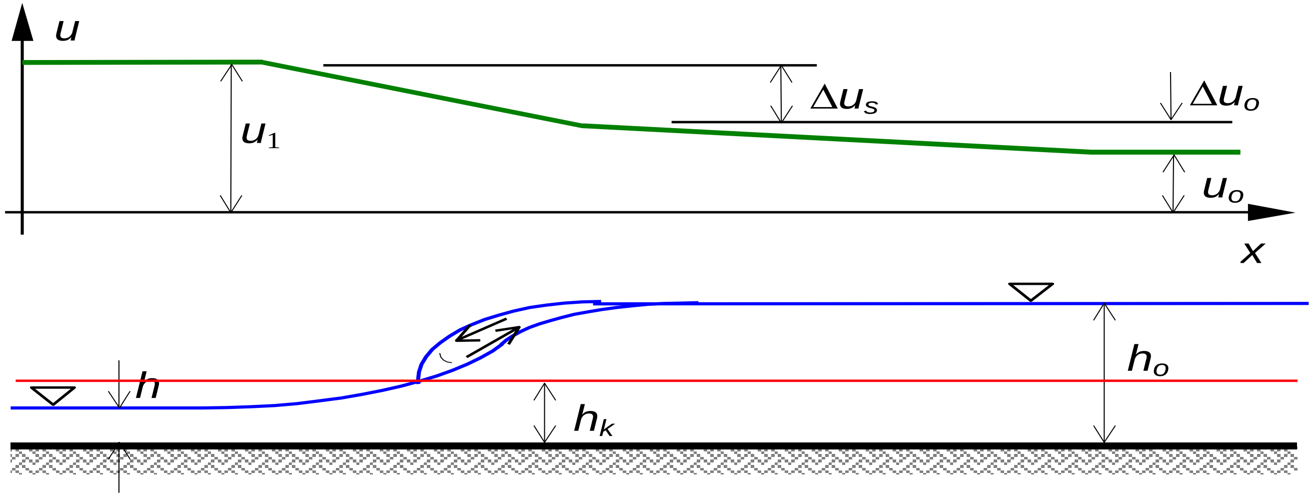


In the lower station, there is a transition from the subcritical depth ***h*** to the normal depth $\boldsymbol{h_o}$ in the outflow channel:

- in the case of supercritical normal depth – by supercritical damming
- in the case of subcritical normal depth – by hydraulic jump

The hydraulic jump is the form taken by the shock wave found in the open channel flow under special conditions.

Hydraulic coupling of two depths



The form and characteristics of the hydraulic jump depend upon both depths *h* *and* h_o – which are known,

- depths: *h* – is determined by the upstream flow condition,

  h_o – is determined by the tail water condition: $h_o = f(S_o, Q, n)$

The form of the hydraulic jump depends on the momentum relation between u_1 and u_o.

Types of hydraulic jumps

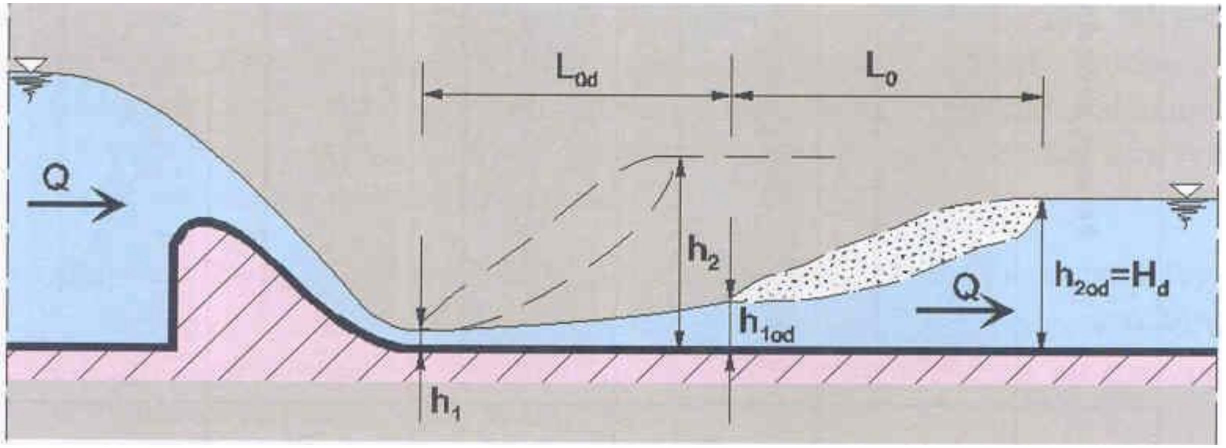
The momentum change Δu is mainly caused by two forces:

- hydrostatic forces – Δu_s
- flow resistance – Δu_o

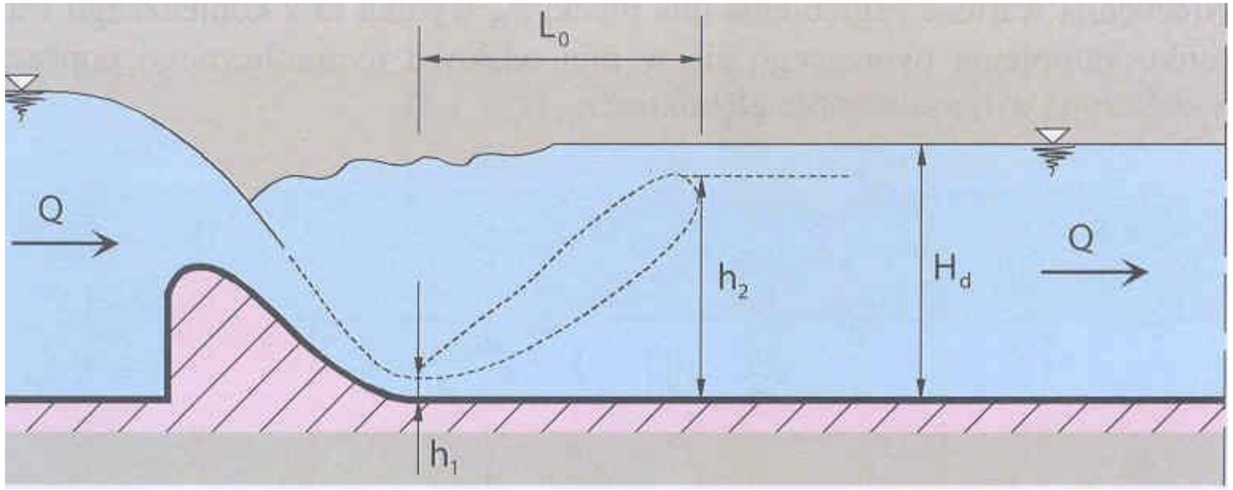
According to the momentum relation, from both sites of the hydraulic jump, one may distinguish:

- free jump
  - ✓ the momentum at the cross-section where smallest depth (h) is larger than at the cross-section of depth (h_o), hence: $\Delta u_o > 0$ – surplus of momentum – which must be reduced by shear stress.
- submerged jump
  - ✓ the momentum at the cross-section where smallest depth (h) is smaller than at the cross-section of depth (h_o), hence: $\Delta u_s < 0$ and submerged jump occurs.

Free and submerged jump



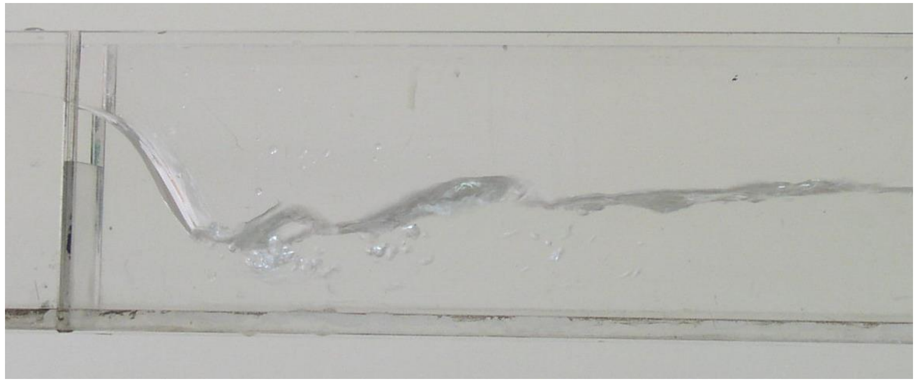
free jump



submerged jump

Hydraulic jump examples

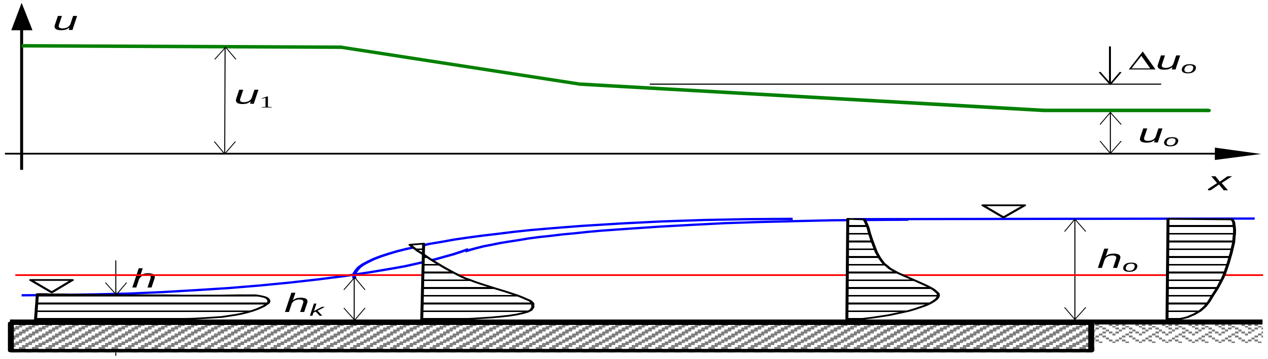
free jump



standing wave

the roller with air entrainment

Free hydraulic jump



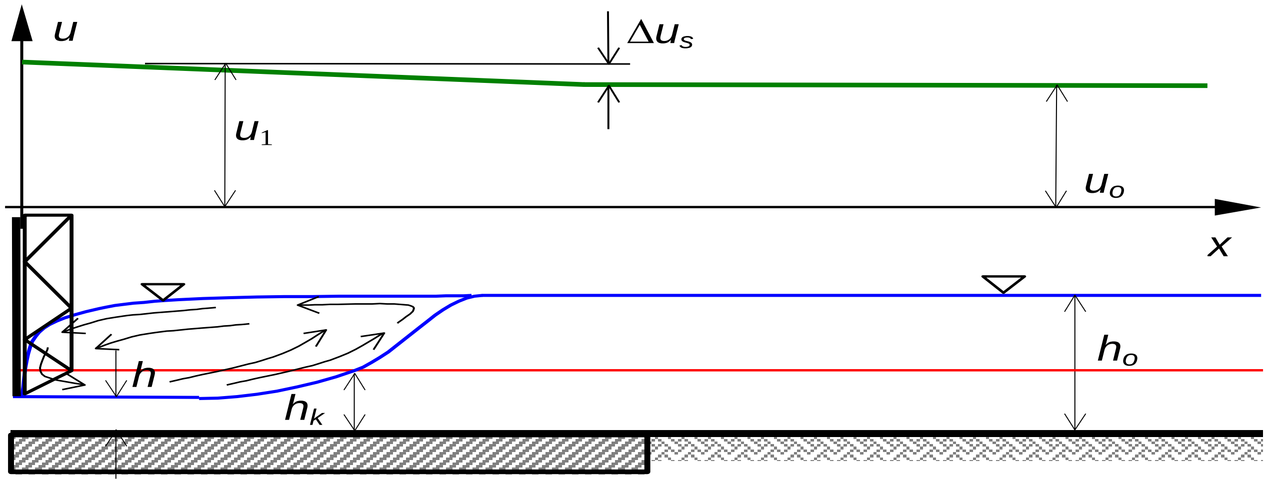
the zone of high bottom velocity that requires heavy bottom protection

low bottom velocity zone

Unsubmerged hydraulic jump should be avoided downstream of the hydraulic structure because:

- the length of the hydraulic jump is large,
- high bottom velocity – requires a heavy bottom protection – a heavily reinforced concrete slab with a thickness even of up to 2 m,
- lack of protection would cause a so-called 'local pothole', threatening the stability of the construct – it can slip, along with the underlying part of the ground, into the pothole; in Łączany, there was a pothole with a depth of 11 m! ($h_o \cong 2$ m).

Submerged hydraulic jump



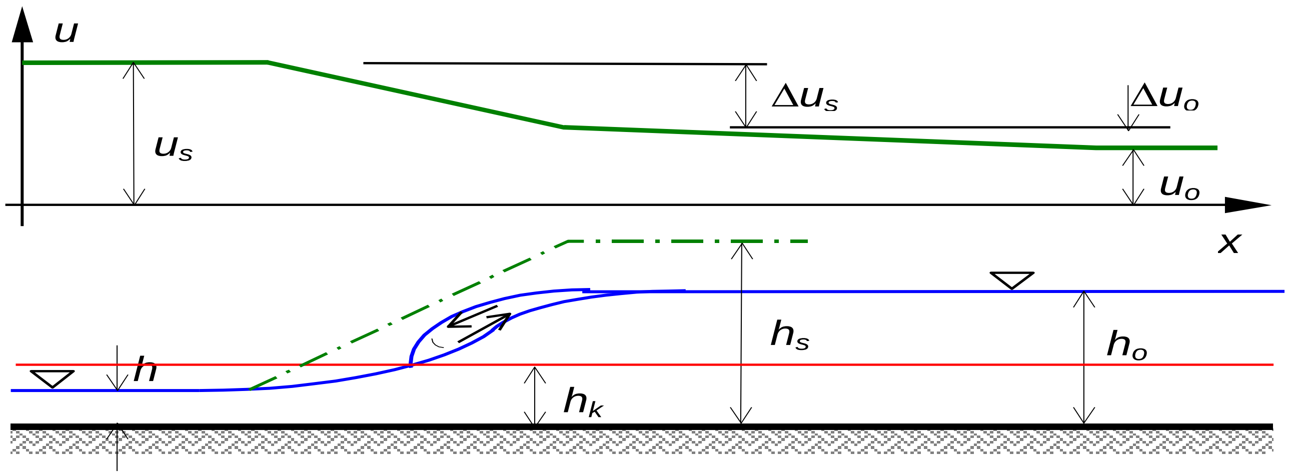
the zone of high bottom velocity that requires heavy bottom protection

low bottom velocity zone

The submerged hydraulic jump is a recommended flow condition downstream of the hydraulic structure because:

- the length of the hydraulic jump is large,
- heavy bottom protection is required for only a short section of the downstream basin (part of the channel).

Analytical criterion of the hydraulic jump submergence



To assess whether the hydraulic jump is submerged, the submergenc criterion is required:

- direct comparison of the momentum flux (Δu_2) after the jump and in the outflow channel is not possible due to its drop during the critical transition (Δu_o),
- it is necessary to use a different function that would allow a comparison between the momentum of streams (u_s and u_o) in the area of the subcritical flow condition zone,
- to facilitate the assessment, it is convenient to calculate a second conjugated depth (h_s) and compare it with tail water depth (h_o).

The momentum conservation equation

The derived momentum equation is based on Newton's 2nd law:

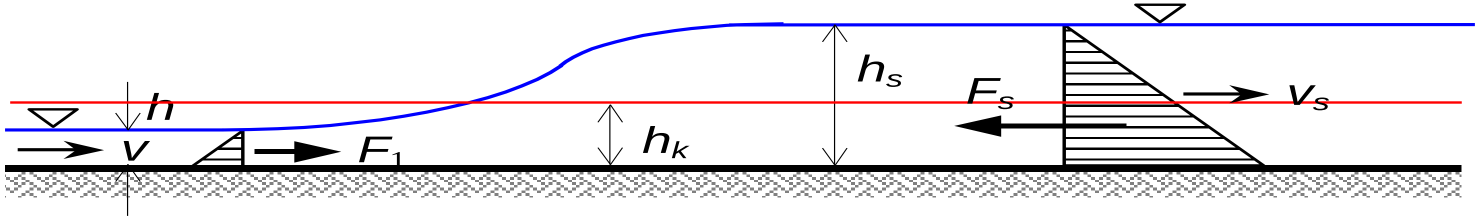
$$F = m\frac{\mathrm{d}v}{\mathrm{d}t} \longrightarrow F\,\mathrm{d}t = m\,\mathrm{d}v \longrightarrow \boxed{F\,\mathrm{d}t = \mathrm{d}u}$$

where: $F\,\mathrm{d}t$ – the force impulse,
$u = m\,v$ – momentum,

hence: the force impulse is the cause of the change of momentum

$$F = m\frac{\mathrm{d}v}{\mathrm{d}t} \equiv \rho V\frac{\mathrm{d}v}{\mathrm{d}t} \equiv \rho\frac{V}{\mathrm{d}t}\mathrm{d}v \equiv \rho Q\,\mathrm{d}v \equiv \rho A\,v\,\mathrm{d}v$$

The conjugated depths relation



The force impulse is caused by the difference of hydrostatic force $F_s - F_1$:

$$F = \gamma \frac{h^2}{2} - \gamma \frac{h_s^{\,2}}{2} = \mathrm{d}(\rho\, A\, v^2) = \frac{\gamma}{g} h_s\, \beta\, v_s^{\,2} - \frac{\gamma}{g} h\, \beta\, v^2$$

where: β – the correction coefficient of momentum (due to averaging of the velocity)

The function of the hydraulic jump:

$$\frac{h^2}{2} + h \frac{\beta v^2}{g} = \frac{h_s^{\,2}}{2} + h_s \frac{\beta v_s^{\,2}}{g} = \mathrm{f}_s(h) = \mathrm{const}$$

The second conjugated depth h_s is achieved by:

$$\boxed{h_s = \frac{h}{2}\left(\sqrt{1 + \frac{8\,\beta\, Q^2}{g\, b^2\, h^3}} - 1\right) = \frac{h}{2}\left(\sqrt{1 + \frac{8\, h_k^{\,3}}{h^3}} - 1\right)}$$

Analytical criterion of the hydraulic jump submergence

The hydraulic jump is submerged when:

$$\eta = \frac{h_o}{h_s} > 1.1$$, where: η – the safety coefficient

If above condition is not satisfied, then it is necessary to apply one or more of the following engineering solutions:

- macro-roughness – increasing of the flow resistance by fixing large stones in the bottom of the basin or baffles – reinforced concrete blocks of various shapes,
- a sill for energy dissipation – overflow weir,
- stilling basin – deepening of the bottom of the outflow channel,
- trampoline – spillway ending with a trampoline and ejecting the stream of water slightly upwards – when it falls, aeration occurs – the fluid becomes more compressible – resulting in less hydrodynamic pressure.

Calculation of the amount of energy dissipated by hydraulic jump

- The energy change:

$$\Delta H = h + \frac{Q^2}{2\,g\,b^2\,h^2} - h_s + \frac{Q^2}{2\,g\,b^2\,h_s^{\,2}}$$

- Momentum equation:

$$\mathrm{f}_s(h) = \frac{h^2}{2} + \frac{\beta\,Q^2}{g\,b^2\,h} = \frac{h_s^{\,2}}{2} + \frac{\beta\,Q^2}{g\,b^2\,h_s}$$

hence: $\dfrac{\beta\,Q^2}{g\,b^2} = \dfrac{h\,h_s\,(h_s + h)}{2}$

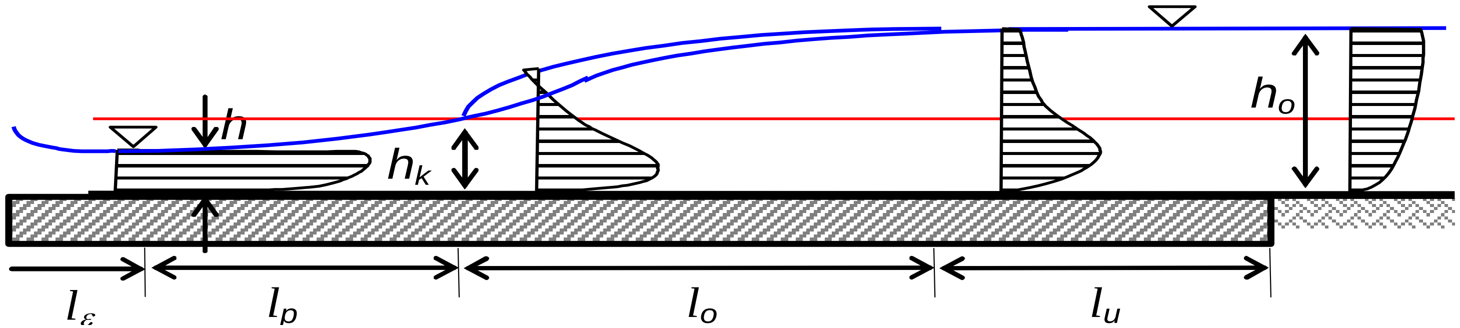
after substitution:

$$\Delta H = h + \frac{h\,h_s\,(h_s - h)}{4\,h^2} - h_s + \frac{h\,h_s\,(h_s - h)}{4\,h_s^{\,2}}$$

finally

$$\boxed{\Delta H = \frac{(h_s - h)^3}{4\,h^2\,h_s^{\,2}}}$$

The length of channel protection



$$l = l_\varepsilon + l_p + l_o + l_u$$

The total length of protection includes:

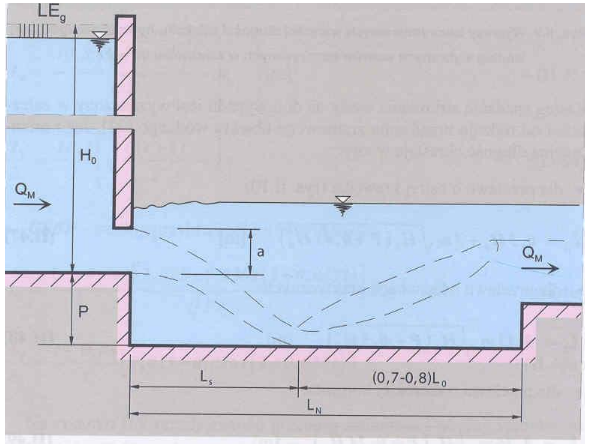
- the length of contraction (l_ε)
- the length of damming (l_p) – supercritical flow condition
- the length of the roller (l_o):

$$l_o = a\, h_s\,, \qquad a = f(h_s / h_o) = 4 \div 6$$

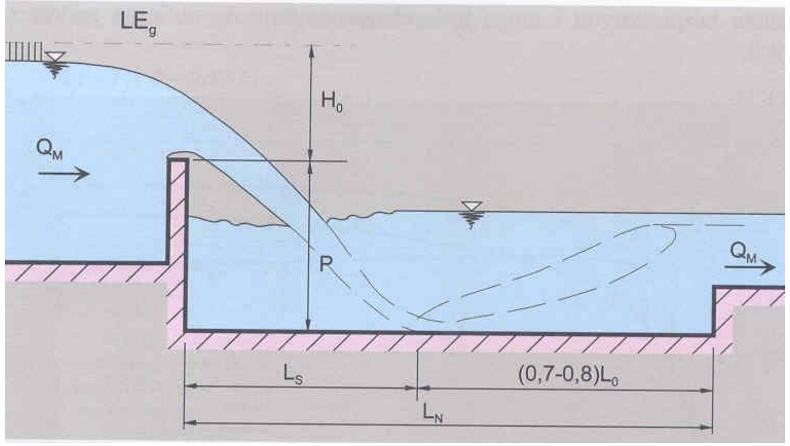
- the length to reach developed velocity distribution (l_u)

$$l_u \cong 15\, h_o$$

Kinetic energy dissipation methods

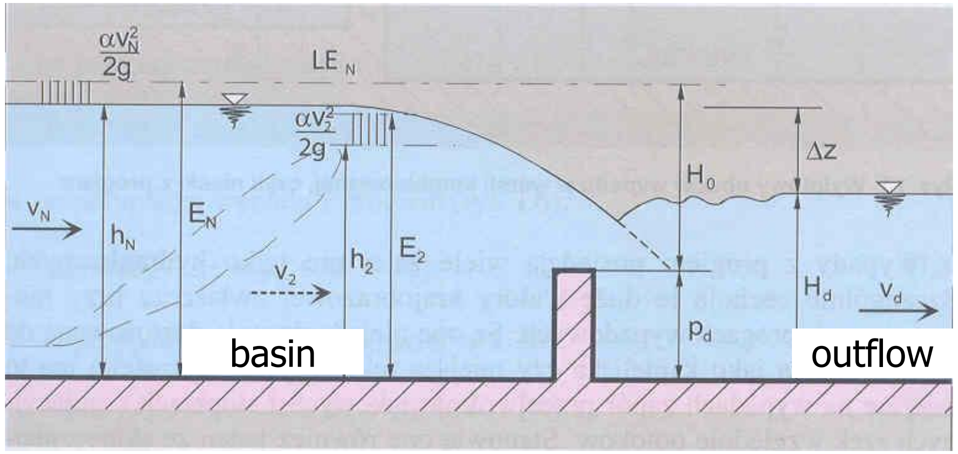


stilling basin for
bottom spillway

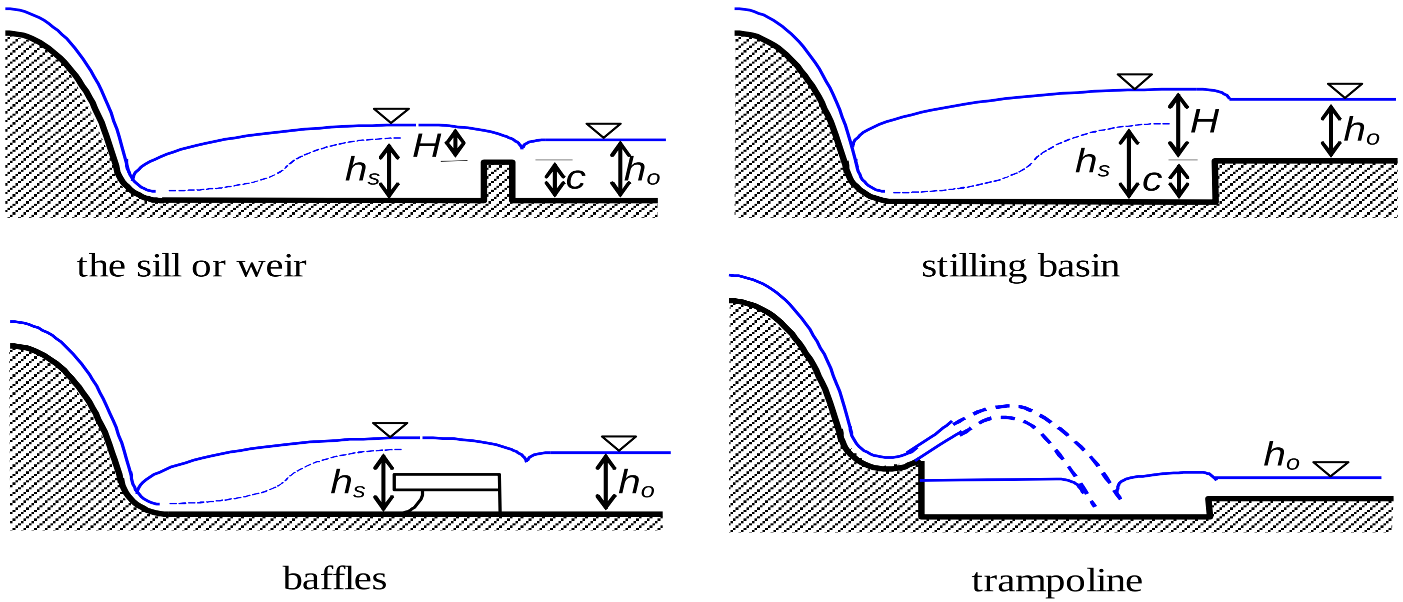


stilling basin for
overflow weir

sill to dissipate
surplus of energy



Designing of the energy dissipators



The hydraulic calculation:

- the sill height calculation:

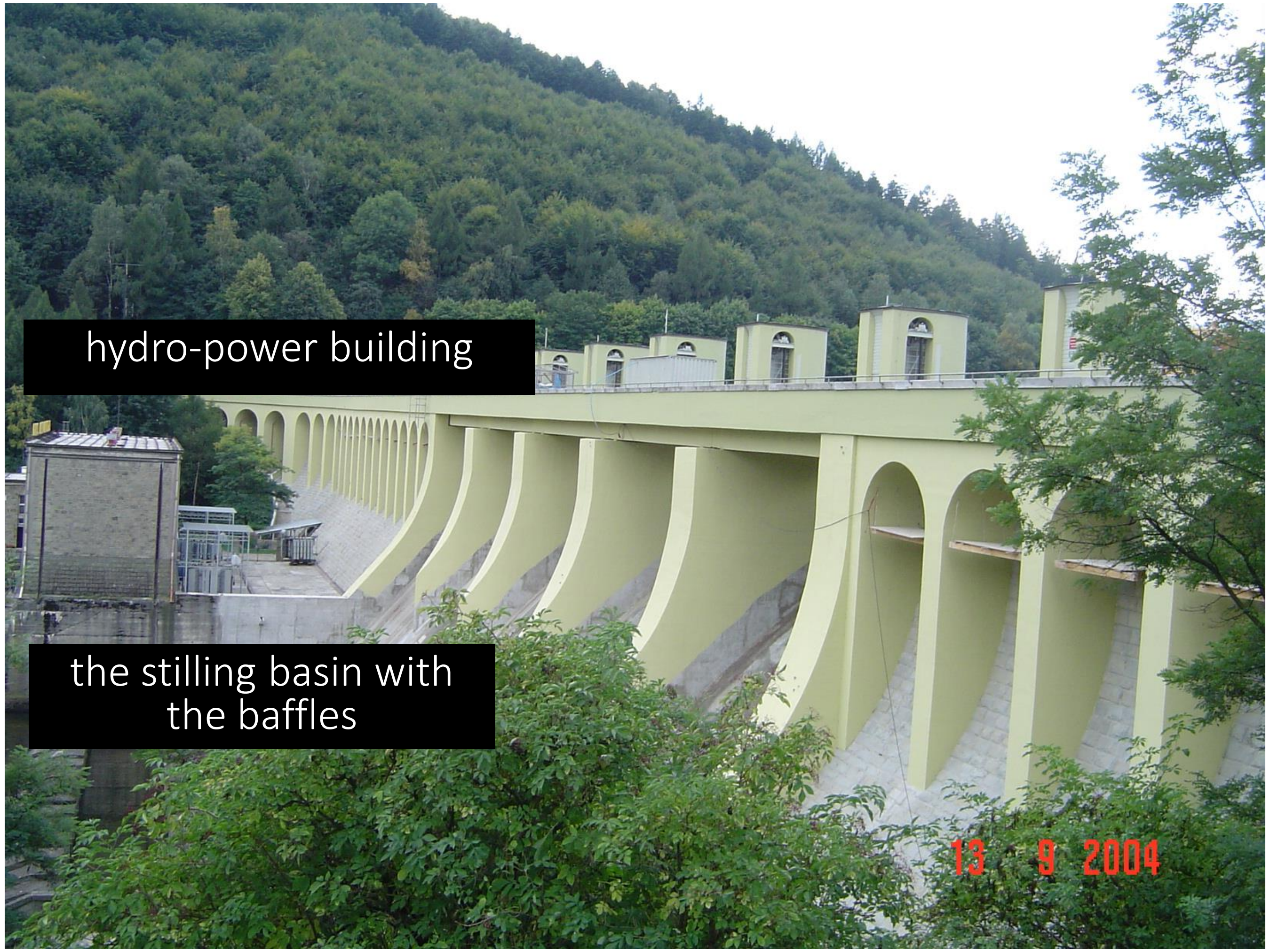
$$Q = \sigma_z \, m \, b \sqrt{2g} \, H_o^{3/2} \;\rightarrow H_o, \; H = H_o - \alpha \, v^2/(2g), \; c = h_s - H$$

- depth of the stilling basin calculation:

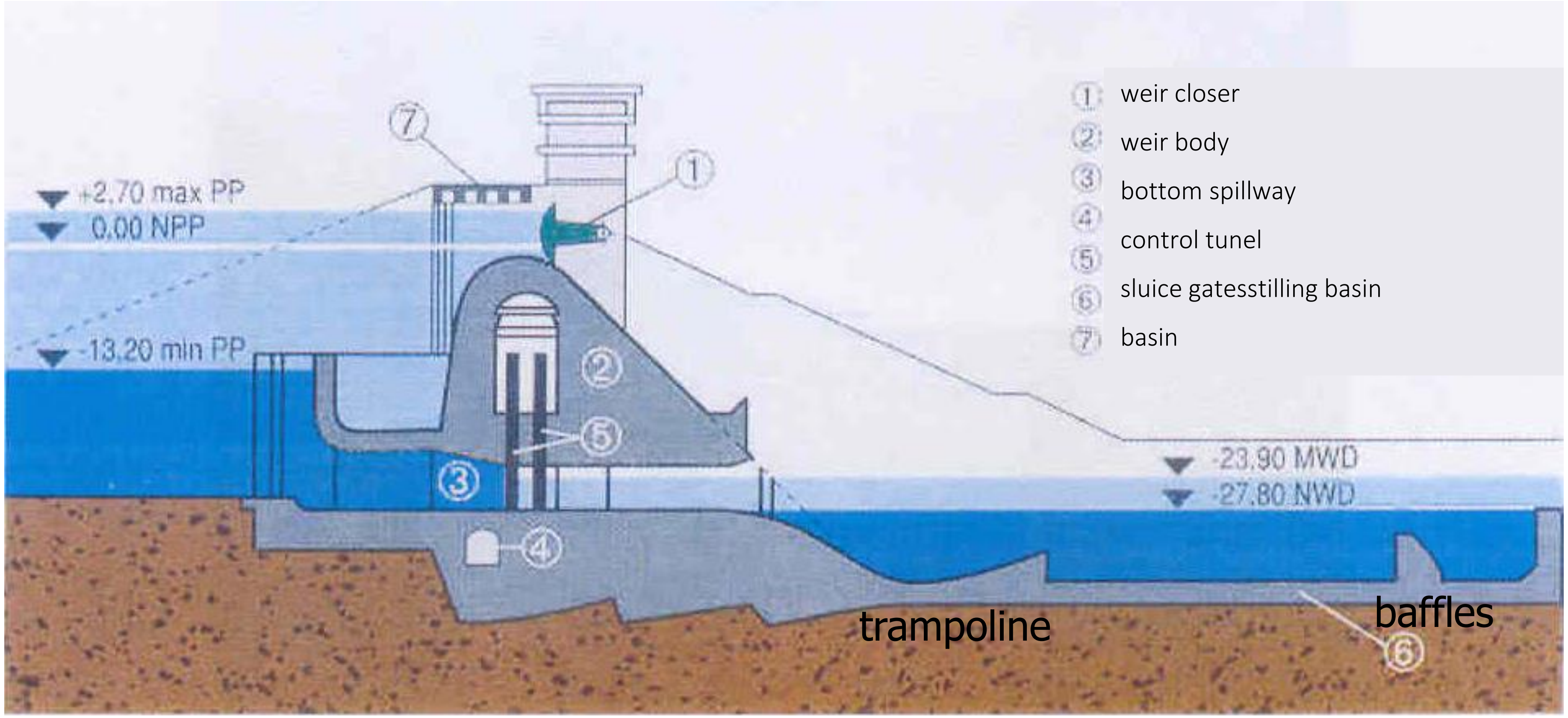
$$\eta = (h_o + c)/h_s \rightarrow c = \eta \, h_s - h_o, \; \eta = 1.05 \div 1.1,$$

Energy dissipators of Tresna dam

Porąbka spillways



Dobczyce spillways and the stilling basin

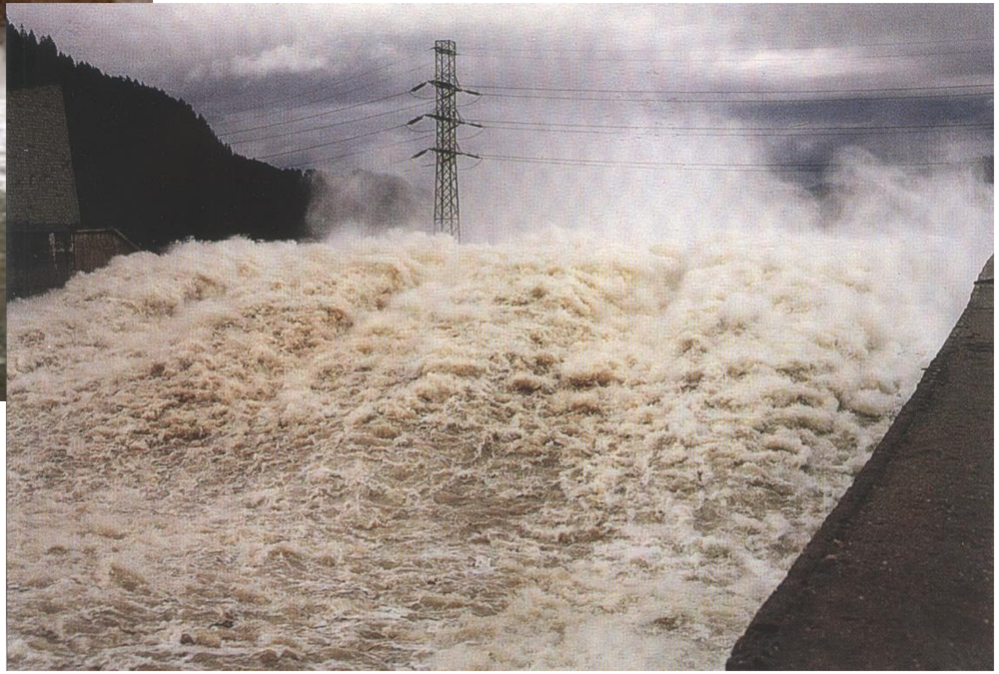


The sill at Grodzisk weir

The trampoline at Czorsztyn dam

the baffles at
trampoline

operation during
flood



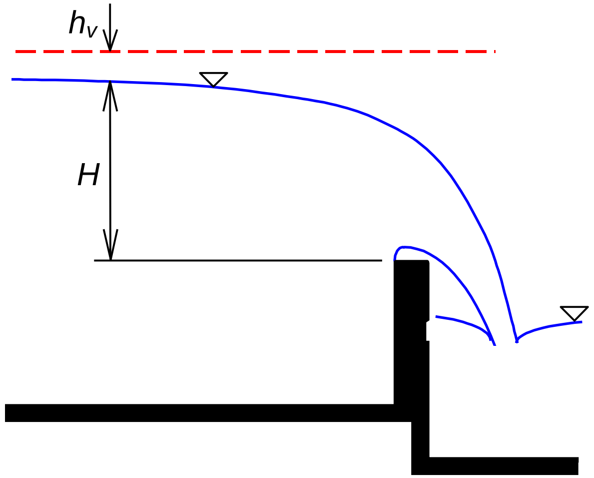
Part 5

Spillways-weirs

Topics

- definition and classification,
- hydraulic design issue,
- ogee shaped weirs,
- submergence criteria.

Definition of weir spillway



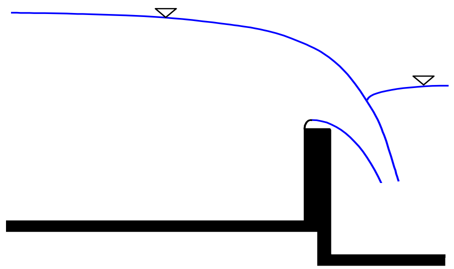
Hydraulic structures used for discharging flow from reservoir into downstream part of river channel.

Hydraulic classification of weir spillways

According to shape:

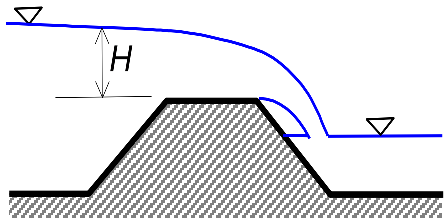
- Sharp crested weirs:

$l \leq 0.1 \div 0.5\ H$



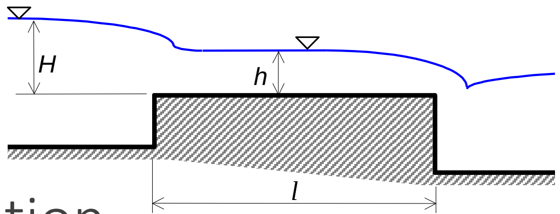
- Ogee shape weirs:

$l \leq 2 \div 2.5\ H$



- Broad crested weirs:

$l \leq 8 \div 15\ H$

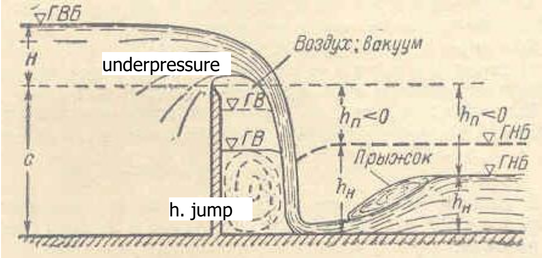


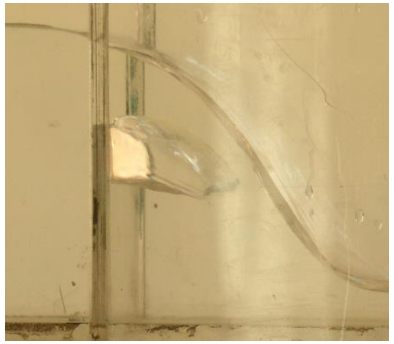
- Dependence on H:

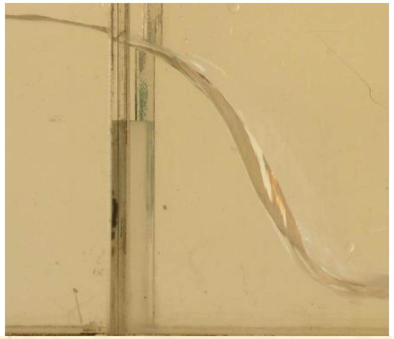
Specification of the weir according to the above classification depends strictly on flow discharge magnitude.

Overflow by sharp crested weir not freely discharged into the air

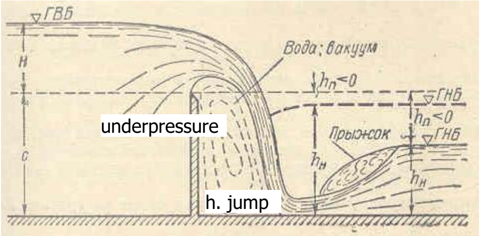
- The under-pressure air zone partly filled with water:
  - ✓ discharge is increasing





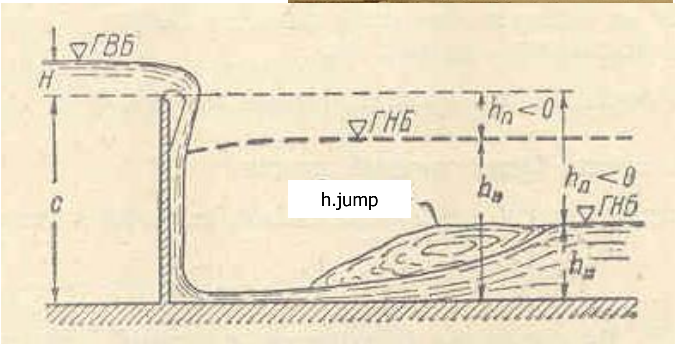


- The under-pressure air zone fully filled with water
  - ✓ larger magnitude of under-pressure

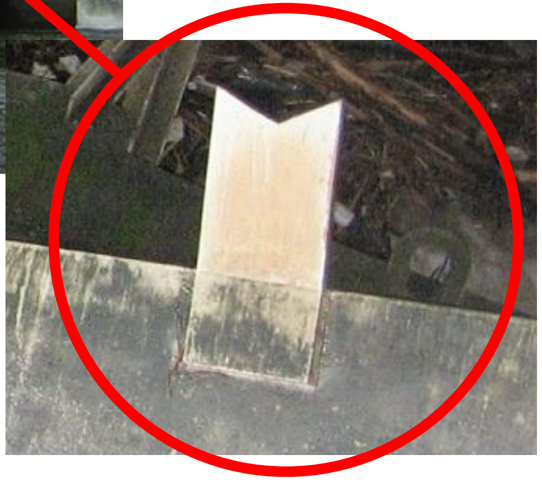


- Stuck stream-flow:
  - ✓ very large under-pressure *m*

Source: Czugajew 1975



Protection from under-pressure



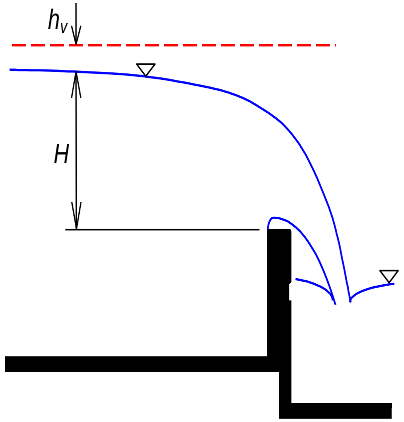
Division of the stream with the help of "swallow tails" ensures the inflow of air under the stream and prevents aspiration and accompanying cavitation of the structure.

Derivation of the formula for calculation of discharge

Unsubmerged weir:

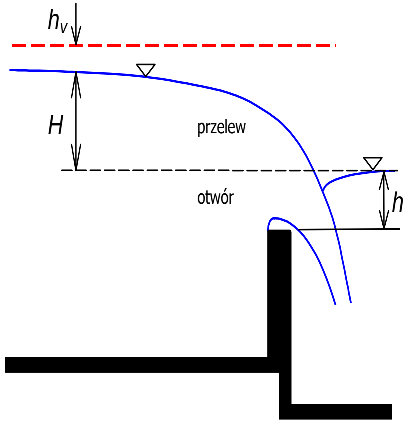
$$\mathrm{d}A = \varepsilon\, b\, \mathrm{d}z \quad \Rightarrow$$

$$\mathrm{d}Q = v\, \mathrm{d}A = b\, \varepsilon\, \varphi \sqrt{2\, g\, z}\, \mathrm{d}z$$



$$Q = \int_{h_v}^{H+h_v} \mathrm{d}Q = \int_{h_v}^{H+h_v} b\, \varepsilon\, \varphi \sqrt{2\, g\, z}\, \mathrm{d}z =$$

$$= \frac{2}{3}\, \mu\, b \sqrt{2\, g} \left[\left(H + h_v\right)^{\frac{3}{2}} - h_v^{\frac{3}{2}} \right]$$



Submerged weir:

The sum of the orifice and free weir discharge

$$Q = \frac{2}{3}\, \mu\, b \sqrt{2\, g} \left[\left(H + h_v\right)^{\frac{3}{2}} - h_v^{\frac{3}{2}} \right] + \mu_z\, b\, h \sqrt{2\, g \left(H + h_v\right)}$$

The simplified formula for calculation of discharge of broad-crested weir

in practice, simplified formulas are used, taking into account the kinetic head h_v and the submergence degree *h* in the empirical coefficients: *m* or *m* and *s*

Unsubmerged weir:

$$\boxed{Q = \frac{2}{3}\mu\, b\,\sqrt{2\,g}\,H_o^{\frac{3}{2}} = m\,b\,\sqrt{2\,g}\,H_o^{\frac{3}{2}}}$$

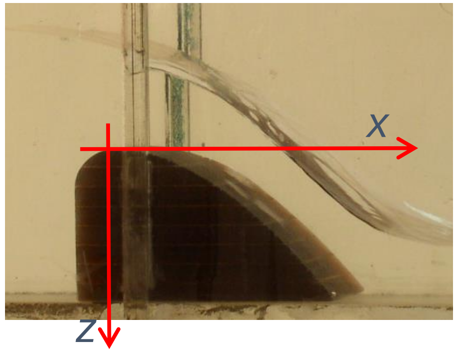
where: $H_o = H + \dfrac{\alpha\, v_o^{\,2}}{2g}$

Submerged weir:

$$\boxed{Q = \frac{2}{3}\sigma\,\mu\, b\,\sqrt{2\,g}\,H_o^{\frac{3}{2}} = m\,\sigma\,b\,\sqrt{2\,g}\,H_o^{\frac{3}{2}}}$$

where: $\sigma\left(\dfrac{h}{H}\right) \in (0;1]$, *H* measured from the crest of weir $H \longrightarrow H + h$

The basic shape function of the ogee weir

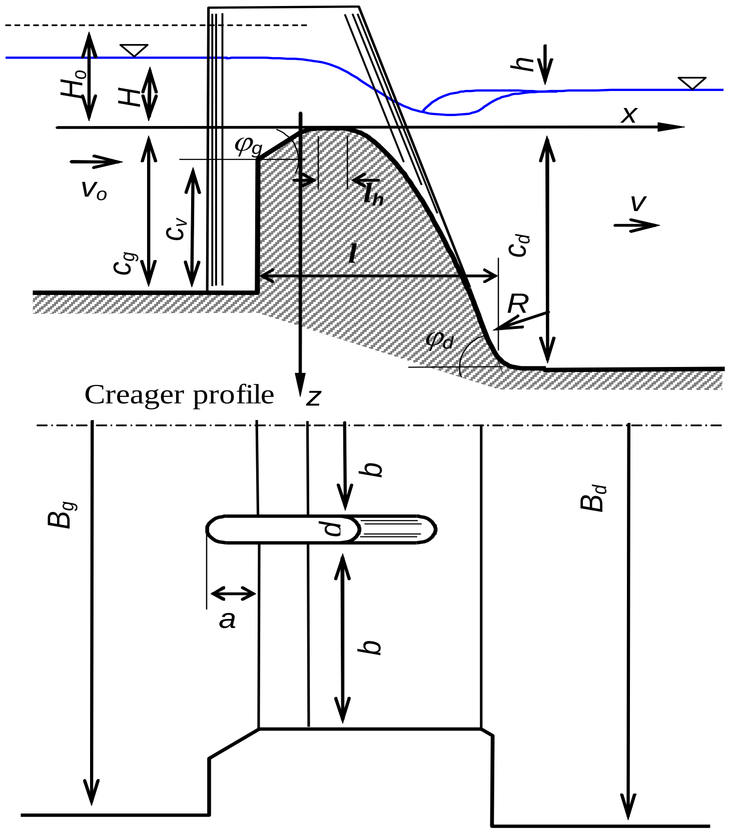


- horizontal velocity: $v_x = \varphi \sqrt{2\, g\, H_o}$
- vertical velocity: $v_z = g\, t$
- the length of free fall: $z = \dfrac{g\, t^2}{2}$
- the time of free fall: $t = \sqrt{\dfrac{2\, z}{g}}$
- the shape of the free fall curve: $x = v_x\, t = \varphi \sqrt{2\, g\, H_o} \sqrt{\dfrac{2\, z}{g}} = 2\, \kappa \sqrt{z\, H_o}$

The rules of weir designing:

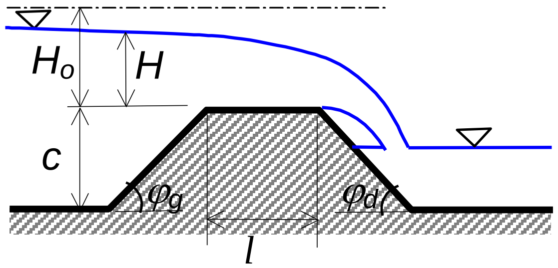
- Shape function is based on the design discharge
- Shape correction is based on the control discharge – taking into account possible underpressure

Practical ogee shaped weirs



- Craeger weir – the profile shape is almost parabolic

Used as relief in damming structures, usually with rectangular shape of outflow cross section.



- Trapezoidal profile of weir

Calculation of the discharge of ogee weir

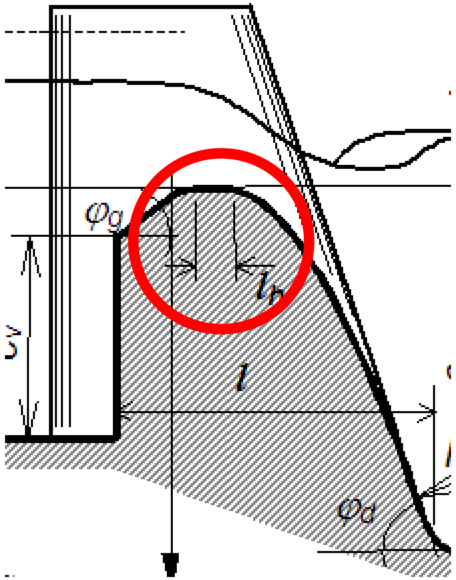
$$Q = m\,\sigma\,b\,\sqrt{2\,g}\,H_o^{3/2} \quad \Rightarrow \quad \boxed{Q = \sigma\,m\,\sigma_k\,\varepsilon\,b\,\sqrt{2\,g}\,H_o^{3/2}}$$

m – discharge coefficient: $m = 2/3\ \mu\ e_v$
σ – submergence coefficient
σ_k – shape correction coefficient
ε – horizontal contraction coefficient (e_h)
b – the width of the weir (the length of the crest line)
H_o – total head according to the crest level

$$H_o = H + \frac{\alpha\,v_o^{\,2}}{2\,g}$$

v_o – velocity at upstream cross section of weir
α – Saint-Venanta coefficient (generally assumed: $a = 1.1$)

The Creager profile discharge coefficient values (m)



Source: Fanti 1972

The case with a straight line entrance

Length lh							
$l_h = 0{,}6\ H$		$l_h = 1{,}2\ H$		$l_h = 1{,}5\ H$		$l_h = 1{,}8\ H$	
l_h / H_1	m	l_h / H_1	m	l_h / H_1	m	l_h / H_1	m
3,000	0,332	6,000	0,341	7,500	0,347	9,000	0,351
1,500	0,361	3,000	0,363	3,750	0,364	4,500	0,365
1,000	0,378	2,000	0,376	2,500	0,374	3,000	0,372
0,750	0,391	1,500	0,386	1,875	0,381	2,250	0,378
0,600	0,401	1,200	0,394	1,500	0,387	1,800	0,383
0,500	0,410	1,000	0,401	1,250	0,391	1,500	0,387
0,428	0,417	0,856	0,406	1,071	0,395	1,285	0,391
0,375	0,424	0,750	0,411	0,933	0,399	1,125	0,393
0,333	0,430	0,666	0,416	0,833	0,402	1,000	0,396
0,900	0,436	0,900	0,419	0,750	0,405	0,900	0,399

The case without a straight line entrance

H_o / c_g	0	0,2	0,4	0,6	0,8
0	0,494	0,491	0,489	0,487	0,485
1	0,483	0,481	0,479	0,477	0,475
2	0,473	0,471	0,468	0,466	0,464
3	0,462	0,460	0,458	0,456	0,454
4	0,452	0,449	0,447	0,445	0,443
5	0,441	0,439	0,437	0,435	0,433
6	0,430	0,428	0,426	0,424	0,422
7	0,420	–	–	–	–

The shape correction coefficient values (s_k)

In the case of the Creager profile

φ_g [°]	φ_d [°]	c_v / c_g 0,0	0,3	0,6	0,9	1,0
15	15	0,880	0,878	0,855	0,850	0,933
	30	0,910	0,908	0,885	0,880	0,974
	45	0,924	0,922	0,899	0,892	0,933
	≥60	0,927	0,925	0,902	0,895	1,000
35	15	0,905	0,904	0,898	0,907	0,933
	30	0,940	0,939	0,932	0,940	0,974
	45	0,957	0,956	0,949	0,956	0,993
	≥60	0,961	0,960	0,954	0,962	1,000
55	15	0,925	0,933	0,922	0,927	0,933
	30	0,962	0,962	0,960	0,964	0,974
	45	0,981	0,981	0,980	0,983	0,993
	≥60	0,985	0,985	0,984	0,989	1,000
75	15	0,930	0,930	0,930	0,930	0,933
	30	0,972	0,972	0,972	0,972	0,974
	45	0,992	0,992	0,992	0,992	0,993
	≥60	0,998	0,998	0,998	0,999	1,000
>75	15	0,933				
	30	0,974				
	45	0,993				
	≥60	1,000				

Source: Fanti 1972

Side contraction effect

When abutments and piers cause side contractions of flow, the effective crest length (L_e) is less than the actual crest length (L). The effective crest length (L_e) can be determined by the following equation:

$$L_e = L - 2\left(N \cdot K_p + K_a\right) \cdot H_e$$

where:

N – the number of piers

H_e – the hydraulic head

K_p – pier contraction coefficient
0. 2 for square-nosed piers with rounded corners;
0.1 for rounded-nosed piers

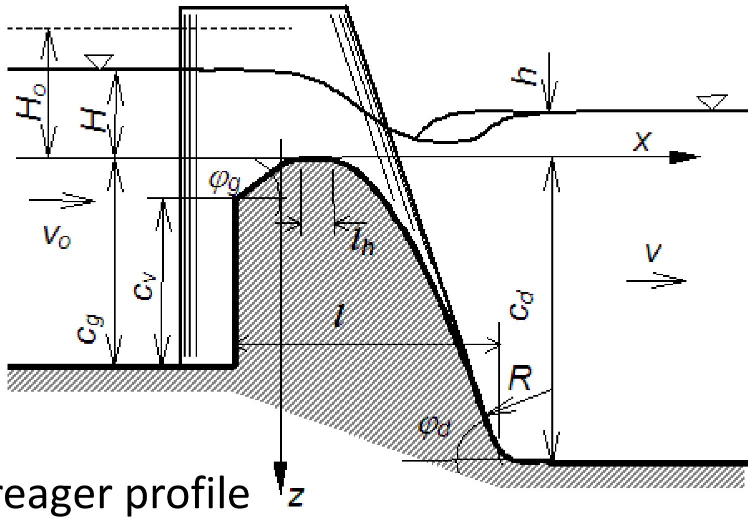
K_a – abutment (end wall) contraction coefficient
0.2 for square abutments with walls 90 degrees to flow direction;
0.1 for rounded abutments ($0.5H_e \le r \le 0.15H_e$)
with walls 90 degrees to flow direction

Criterion of the weir submergence

The following relations must be satisfied:

- $h > 0$
- $h > 0.4\,H$
- hydraulic jump will not occur if:

$$\frac{H-h}{c_d} \le \left(\frac{H-h}{c_d}\right)_k$$



Creager profile

The required degree of submergence to reduce discharge of weir by tailwater $\left(\frac{H-h}{c_d}\right)_k$

m	H / c_d									
	0,10	0,20	0,30	0,40	0,50	0,75	1,00	1,50	2,00	3,00
0,42	0,89	0,84	0,80	0,78	0,76	0,73	0,73	0,76	0,82	1,00
0,46	0,88	0,82	0,78	0,76	0,74	0,71	0,70	0,73	0,79	1,01
0,48	0,86	0,80	0,76	0,74	0,71	0,68	0,67	0,70	0,78	1,02
0,49	0,86	0,80	0,76	0,73	0,70	0,67	0,66	0,70	0,78	0,99

Submergence coefficient values

Submergence coefficient (σ)

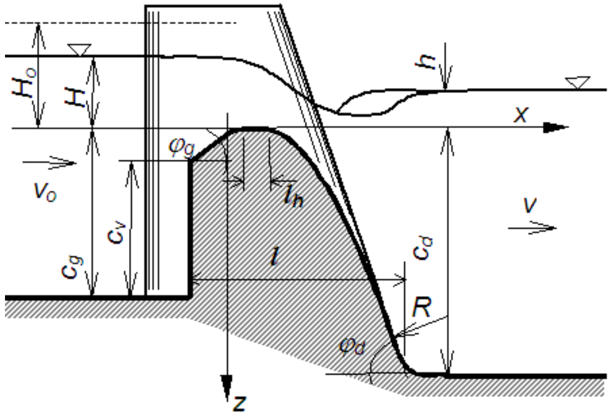
h/H_o	σ	h/H_o	σ
0.00	1.00	0.55	0.965
0.05	0.999	0.60	0.957
0.10	0.998	0.65	0.947
0.15	0.997	0.70	0.933
0.20	0.996	0.75	0.910÷0.800
0.25	0.994	0.80	0.760
0.30	0.991	0.85	0.700
0.35	0.988	0.90	0.590
0.40	0.983	0.95	0.410
0.45	0.978	1.00	0.000
0.50	0.972	–	–

Source: Fanti 1972

The practical shape function of the Creager weir

- coordinates of the profile (*Fanti 1972*):

  H_o – the designing total head

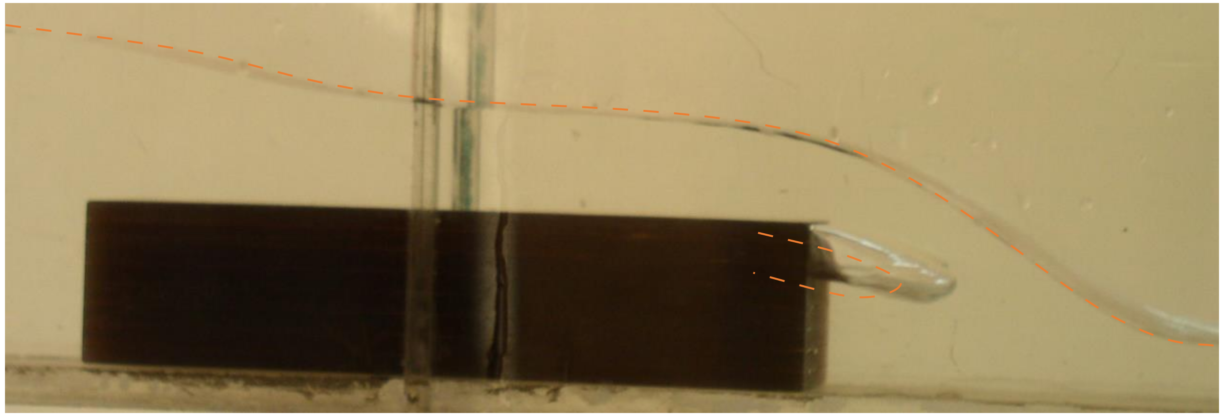


- radius of curvature – *R*:

x/H_o	z/H_o	x/H_o	z/H_o	x/H_o	z/H_o
0	0,126	1,4	0,564	2,8	2,462
0,1	0,036	1,5	0,661	2,9	2,640
0,2	0,007	1,6	0,764	3,0	2,824
0,3	0,000	1,7	0,873	3,1	3,013
0,4	0,006	1,8	0,987	3,2	3,207
0,5	0,027	1,9	1,108	3,3	3,405
0,6	0,060	2,0	1,235	3,4	3,609
0,7	0,100	2,1	1,369	3,5	3,818
0,8	0,146	2,2	1,508	3,6	4,031
0,9	0,198	2,3	1,653	3,7	4,249
1,0	0,256	2,4	1,894	3,8	4,471
1,1	0,321	2,5	1,960	3,9	4,698
1,2	0,394	2,6	2,122	4,0	4,930
1,3	0,475	2,7	2,279	4,5	6,220

c [m]	*H* [m]								
	1,0	2,0	3,0	4,0	5,0	6,0	7,0	8,0	9,0
10	3,0	4,2	5,4	6,5	7,5	8,5	9,6	10,6	11,6
20	4,0	6,0	7,8	8,9	10,0	11,0	12,2	13,3	14,3
30	4,5	7,5	9,7	11,0	12,4	13,5	14,7	15,8	16,8
40	4,7	8,4	11,0	13,0	14,5	15,8	17,0	18,0	19,0
50	4,8	8,8	12,2	14,5	16,5	18,0	19,2	20,3	21,3
60	4,9	8,9	13,0	15,5	18,0	20,0	21,2	22,2	23,2

Broad crested weir



Shape of water profile in the case of unsubmerged weir

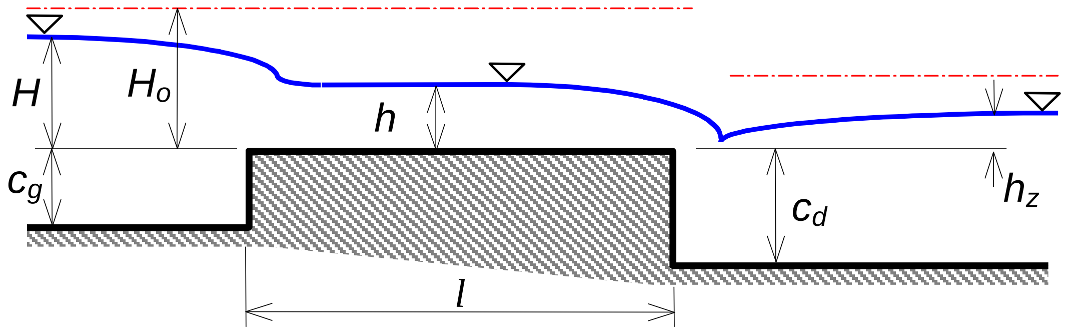
Shape of water profile in the case of submerged weir

Broad crested weir

1. Discharge calculation

Water profile shape influenced by:

- local head loss at the entrance
- almost uniform flow over crest
- local head loss at the outlet



Basic equation:

- Bernoullie equation: $H_o = h + \frac{v^2}{2g} + \zeta_w \frac{v^2}{2g}$
- continuity equation: $Q = b_p\, h\, v$
- velocity coeficient φ: $\varphi = \frac{1}{\sqrt{1+\zeta_w}}$

Flow discharge formula:

$$\boxed{Q = b\,h\,\varphi\,\sqrt{2\,g\,(H_o - h)}}$$

Determination of water depth (*h*) on the crest:

- according to Balanger:
  $q = \max \longrightarrow h = h_k = 2/3\ H_o$

- $\longrightarrow m = 0.385\ \varphi \qquad \longrightarrow \qquad \boxed{Q = m\,b\,\sqrt{2\,g}\ H_o^{3/2}}$

generally used simplification

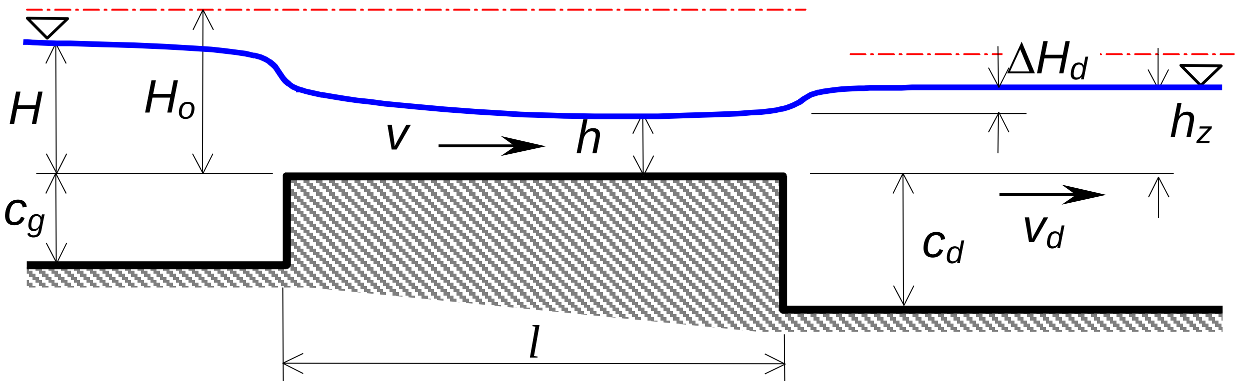
- according to Bachmietiew:

$$m = \sqrt{\frac{k^3}{2}} \qquad k = \frac{2\,\varphi^2}{1 + 2\,\varphi^2}$$

- according to experimental data:
  $h = k\ H_o$, $k(m = 2/3\ \mu\ \varepsilon) \in [0.4; 0.6]$ at $m \in [0.3; 0.38]$,

Broad crested weir

2. Criterion of weir submergence



- classical approach:

  $h_z > h$, $\Delta H_d \cong 0 \rightarrow h > 2/3\ H_o$,
  or $h_z > h_k$

- empirical formula:

  unsubmerged weir on the crest, $h_z > n\ H_o$, $n \in [0.75;0.85]$

Broad crested weir

3. Discharge of the submerged weir

- empirical formula:

$$\boxed{Q = b\,h\,\varphi_z\,\sqrt{2\,g\,(H_o - h)}}$$

where: $h = h_z - \Delta H_d$ – depth at the crest

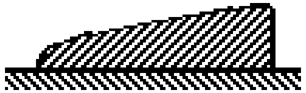
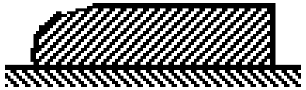
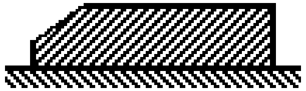
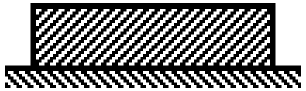
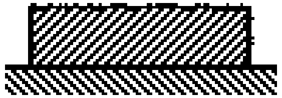
$\Delta H_d(h_k, h_z, b, B_o, c_d)$

- classical formula:

$$\boxed{Q = b\,h_z\,\varphi\,\sqrt{2\,g\,(H_o - h_z)}}$$

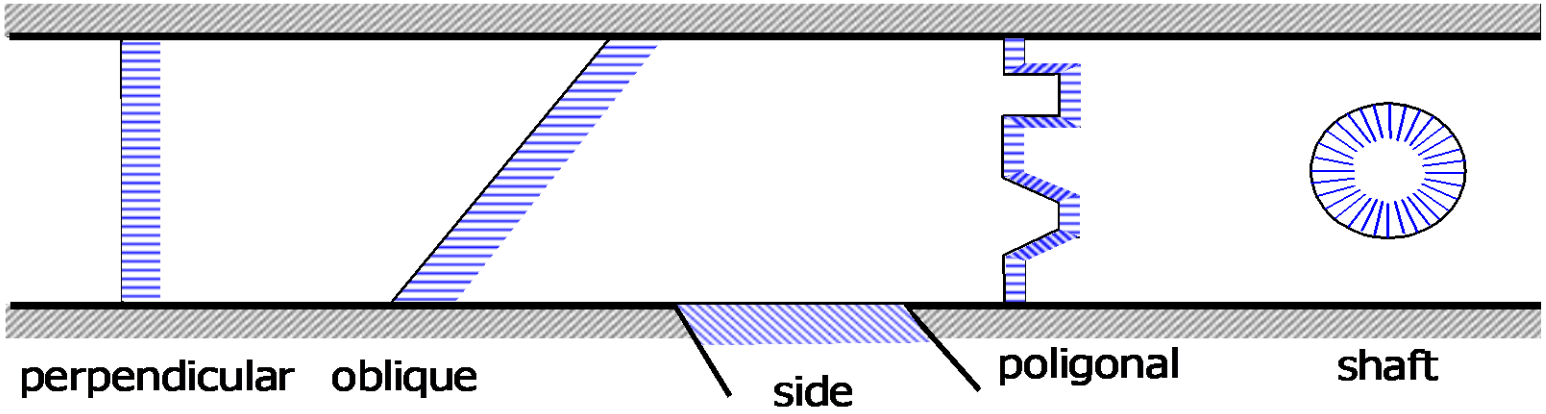
for trapezoidal shape of the weir, the following values of coefficient are assumed: $m = 0.32$, $k = 0.59$, $\varphi = 0.85$

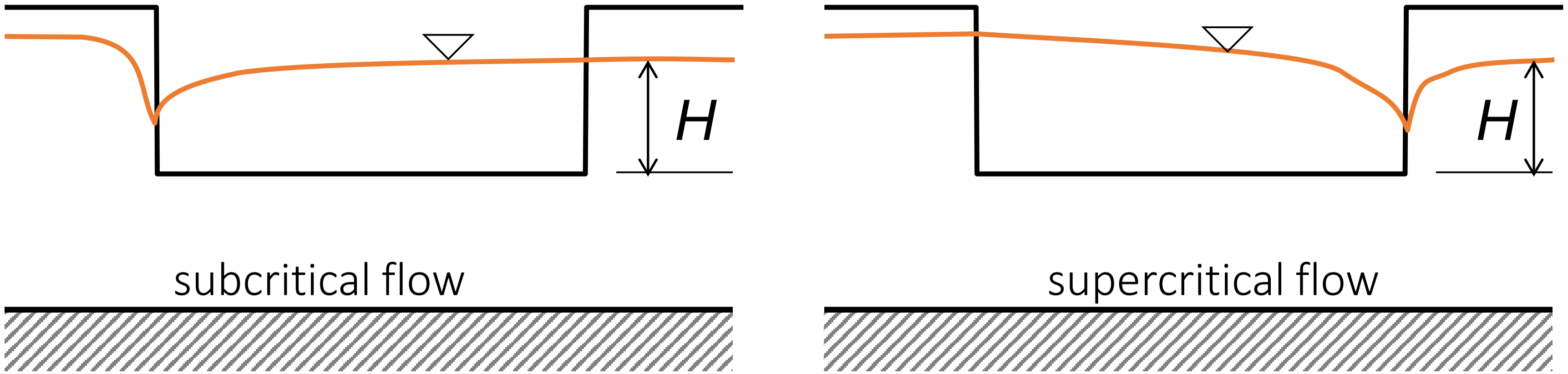
Coefficient values

Shape	ϕ	*k*	*m*
theoretical	1	2/3	0.387
	0.95	0.645	0.365
	0.92	0.63	0.35
	0.88	0.61	0.335
	0.85	0.59	0.32
	0.8	0.56	0.295

Source: Czetwertyński 1958

Weir classification according to planned position





the water-table shape on the side weir

Calculation of discharge of polygonal weir

Polygonal weir:

$$Q = m\left(\Sigma b + \Sigma \sigma_u\, b_u\right)\sqrt{2g}\, H_o^{3/2}$$

where: b, b_u – lengths of perpendicular and oblique parts

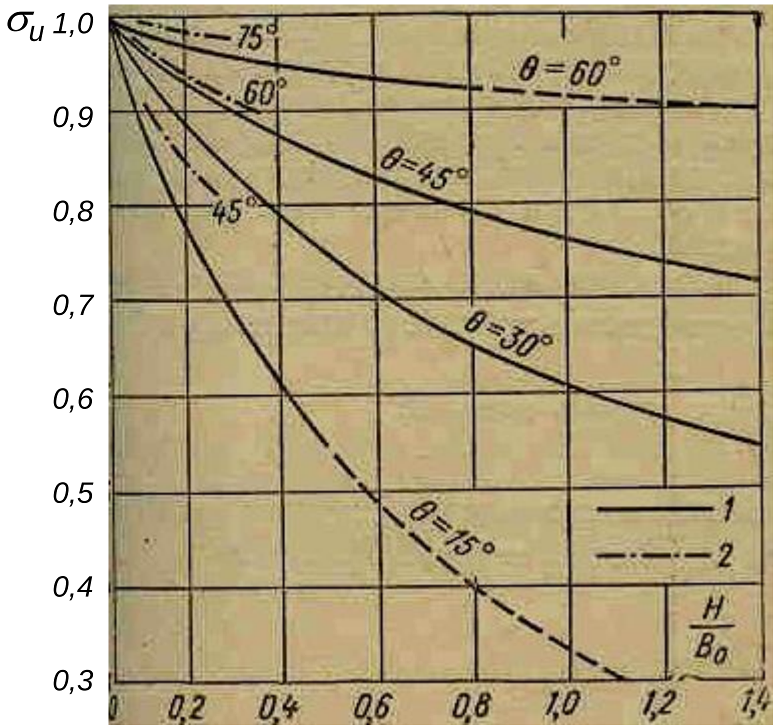
Oblique weir:

$$Q = \sigma_u\, m\, b\, \sqrt{2g}\, H_o^{3/2}$$

where: σ_u – coefficient

1 – sharp crested weir,

2 – ogee weir



Source: Czugajew 1975

Calculation of discharge of side weir

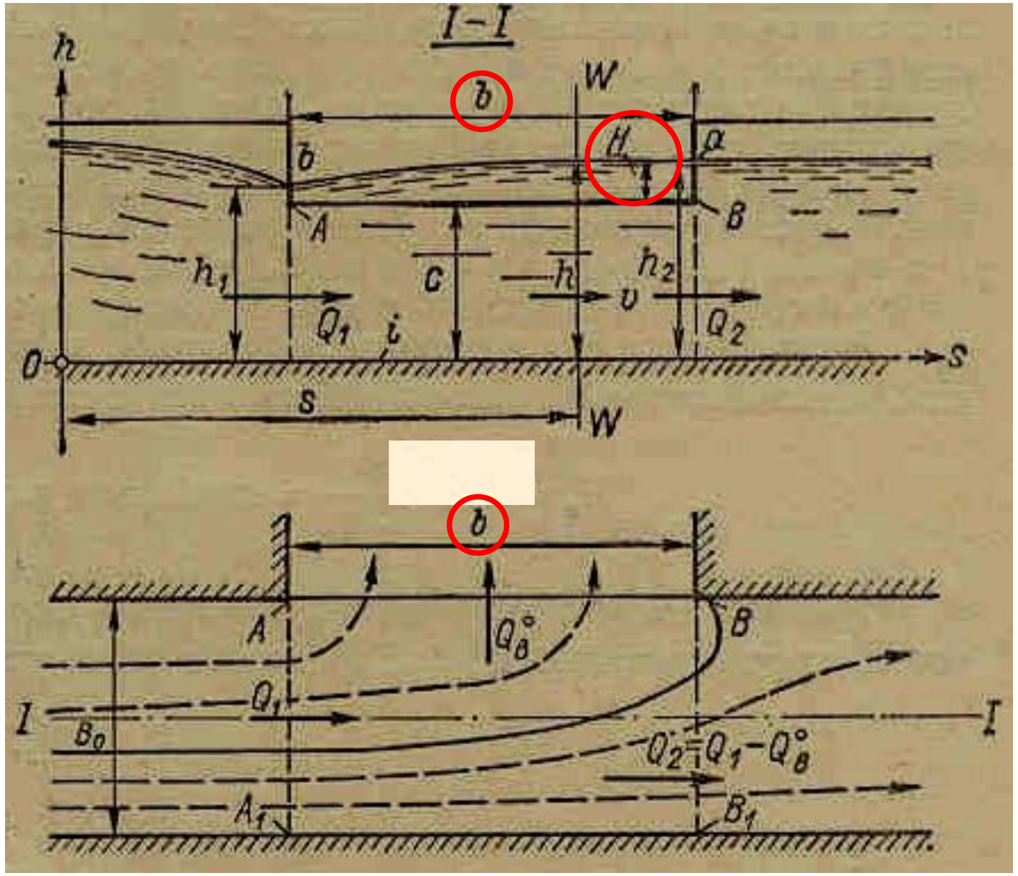
Flow discharge formula:

$$Q = \sigma_b \, m \, b \sqrt{2g} \, H_o^{3/2}$$

where: σ_b – coefficient

$$m = \tfrac{2}{3}\mu = \tfrac{2}{3} \times 0.83 = 0.553$$

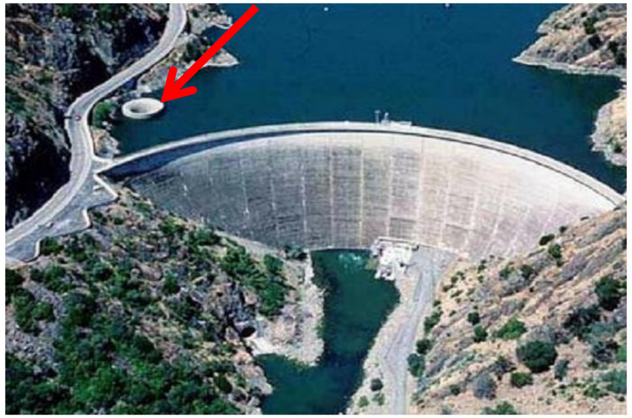
$$\sigma_b = \sqrt[6]{\frac{H}{b}}$$



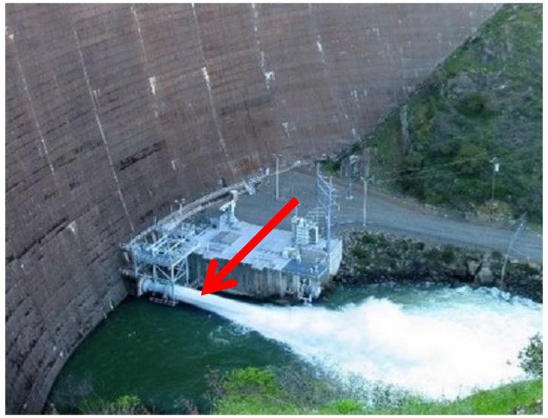
Source: Czugajew 1975

Shaft weirs

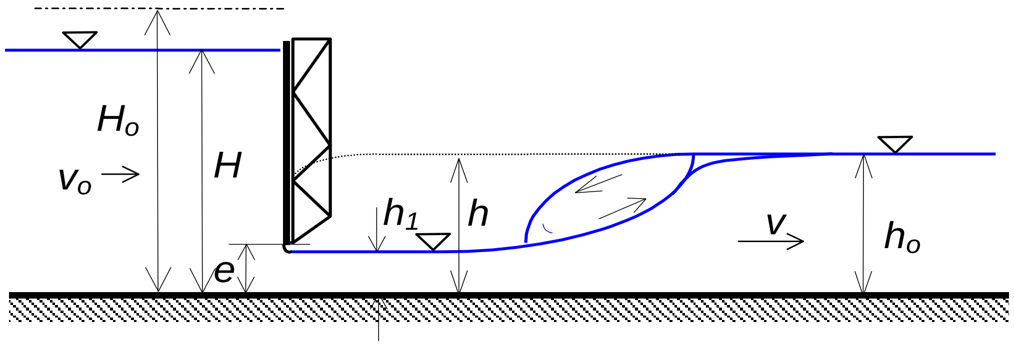
Shaft weir in Monticello (California)



Inlet and outlet of the shaft weir Monticello



Sluice gate outflow



Unsubmerged outflow:

$h < e,\ h_1 = \varepsilon\, e$

$$Q = \mu\, b\, e \sqrt{2\, g\, H_o}$$

Submerged outflow:

$h > h_{critical}$

$$Q = \mu\, b\, e \sqrt{2\, g\, (H_o - h)} = \varphi\, b\, h \sqrt{2\, g\, (H_o - h)}$$

$$Q = \mu\, b\, e \sqrt{2\, g\, (H_o - h_o)}$$

Parameters of flow formula

Formula for depth (h) at the outflow cross-section of gate based on tail water depth:

$$h = \frac{M}{2} + \sqrt{h_o^{\,2} - M \cdot \left(H_o - \frac{M}{4} \right)}, \text{ where: } M = \frac{4 \cdot \mu \cdot e \cdot (h_o - \mu \cdot e)}{h_o}$$

coefficient of discharge μ

$\frac{e}{H_o}$	μ	$\frac{e}{H_o}$	μ	$\frac{e}{H_o}$	μ
0.00	0.617	0.30	0.629	0.55	0.639
0.10	0.621	0.35	0.631	0.60	0.641
0.15	0.623	0.40	0.633	0.65	0.643
0.20	0.625	0.45	0.635	0.70	0.645
0.25	0.627	0.50	0.637	–	–

vertical contraction:

- unsubmerged condition: $h = \varepsilon\, e$, $\varepsilon = 0.62 \div 0.64$, (*hydraulic jump occurs*)
- submerged condition: for $e > (0.15 \div 0.20)\, H_o$, or $h_o > 2.5\, e$, *there is no contraction*, $\varepsilon = 1$

Part 6

Hydraulics of bridges and culverts

Topics

- classification,
- hydraulic design issues,
- structure interference on the flow condition,
- the minimum opening width calculation.

Classification

- Legislation background
  Regulation of MTiGM dated 30 V 2000 on technical conditions to be met by road engineering facilities and their location; Dz. U. 63, 3 VIII 2000
- Classification of bridges and culverts
  - **large bridge** – b (total width) > 10 m:
    - ✓ with erodible riverbed in the whole channel,
    - ✓ with erodible riverbed in the river current,
    - ✓ with unerodible river bed.
  - **small bridge**:
    b < 10 m, unerodible river bed or protected
  - **culvert** (def: a structure that allows water to flow under a road, railroad)
    protected riverbed, pressure flow is allowed.

Basic concepts

Designed flow volumetric rate

- the peak discharge of given probability of exceedance: Q_m *probability value (m) depends on the type of the road,*
- bridges: Q_m – 0.3 do 3%,
- culverts: Q_m – 1 do 5%.

The minimum acceptable total width of bridge opening

- b = S b_i ,
- river type ~ *Q1%/Q50%*,
- mountain rivers: (additional opening increase) *b + 15%*,
- lowland rivers,
- additional requirements for mountain rivers,
- embankment space > 1.5 m,
- 1 span width > 25 m (woody debris are the reason for this req.),
- without piers within the river current.

Basic concepts

Elevation of the structure above the water table

- bridge:
  - roadway: 0.5 m,
  - railway: 0.6 m,
- culvert:

0.7 m from the road way crest, submergence degree: $\Delta H \leq 0.2 \div 0.3$ m.

Influence of bridge types on flow condition

Bridges that do not interfere with the flow

- supporting structure entirely outside the channel

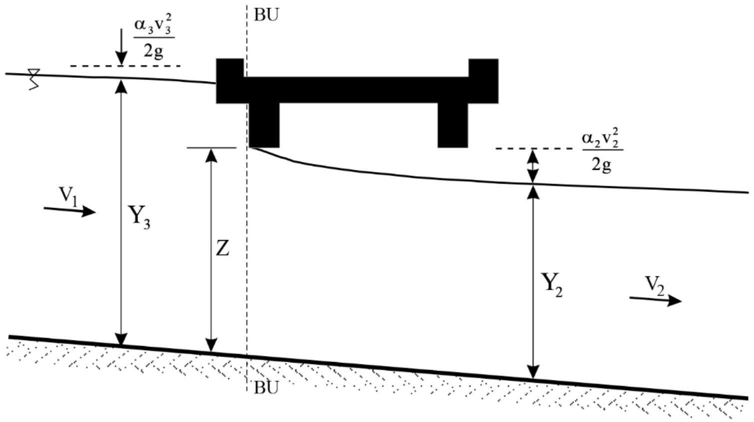
Bridges that interfere with the flow

- bridgeheads and piers narrow the bridge opening

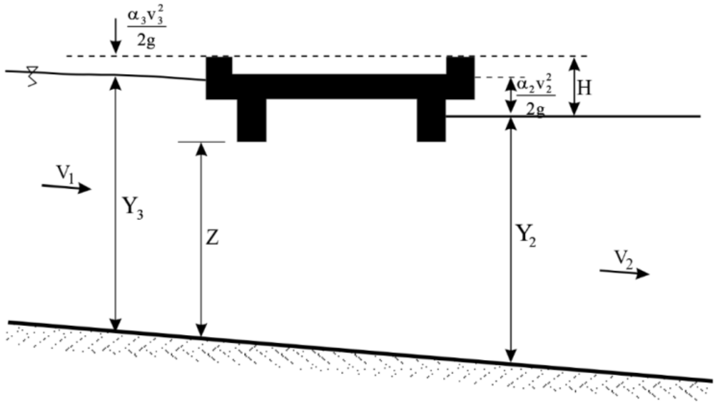
Low water-table level bridges

- not adapted to pass the design flow rate – they cause water to build up above the bridge structure

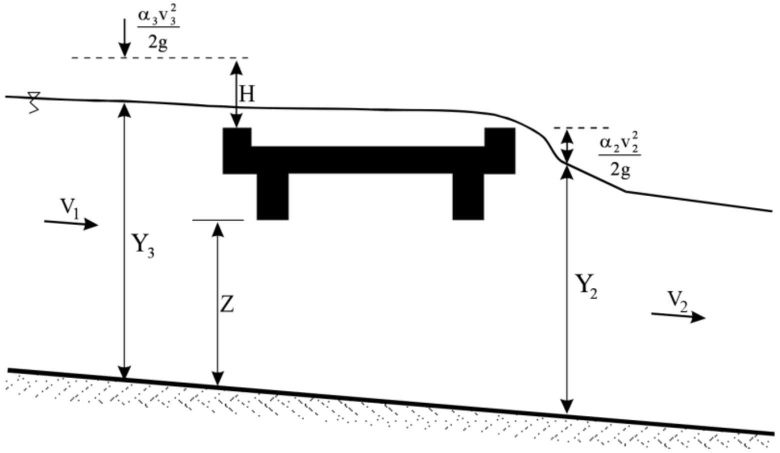
Low water-table level bridges



Submerged entrance



Pressure flow below the bridge

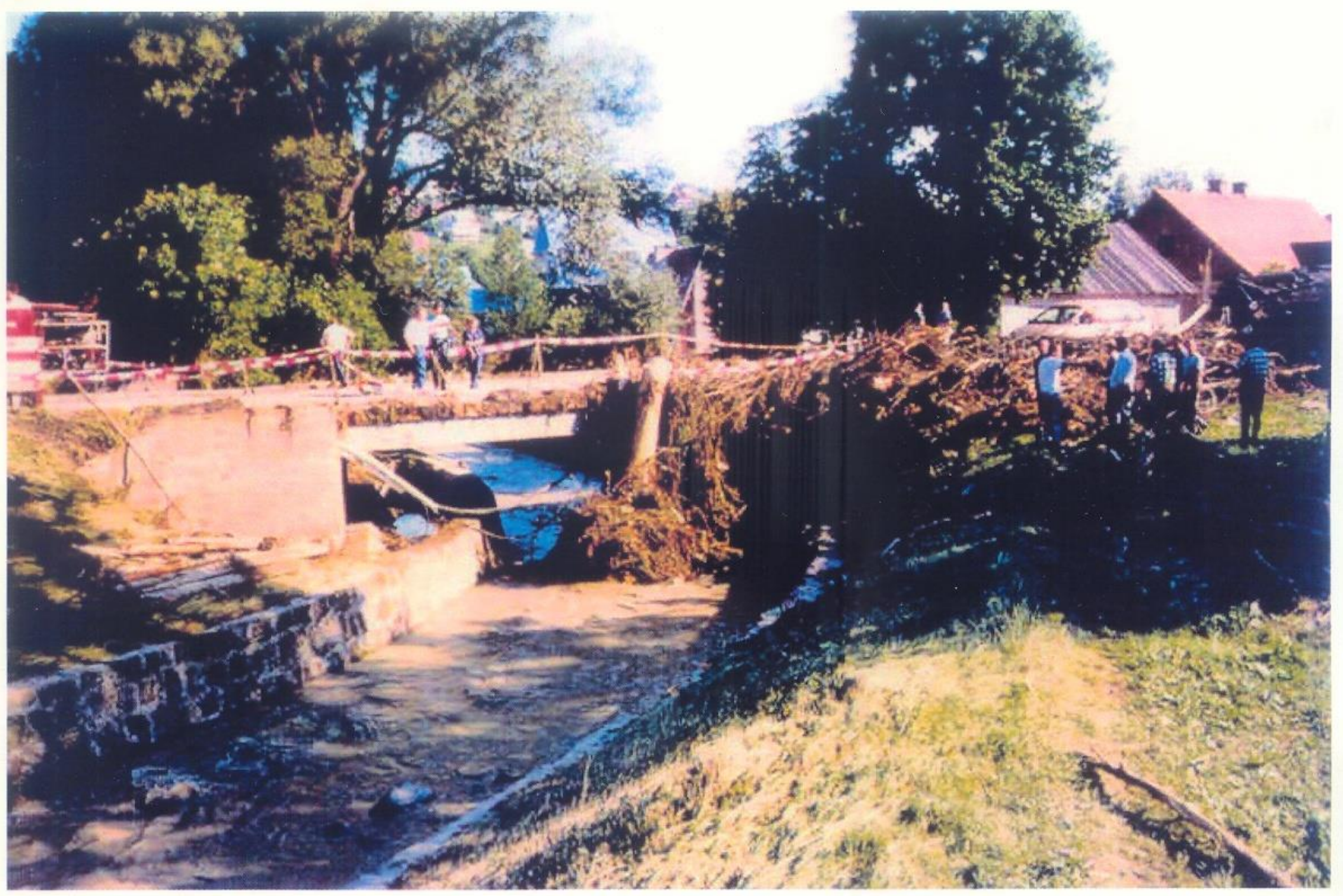


Overflow of bridge and pressure flow below the bridge

The Dębnicki bridge during flood flow




Damaged bridge



Hydraulic calculation

Conditions for passing water under the bridge :

- Non-erodible velocity of the flow $v < v_d$,
  where: v_d – minimum erodible velocity,
- subcritical slope of the channel bed: $S_o < S_k$,
- total head (energy) is not increased due to bridge structure interfering with flow:
  H = const.

Flow regime according to Froude number:

- large bridge – only subcritical flow condition allowed,
- small bridge – acceptable bridge damming.
  (pressure flow acceptable)

Mountain culverts:

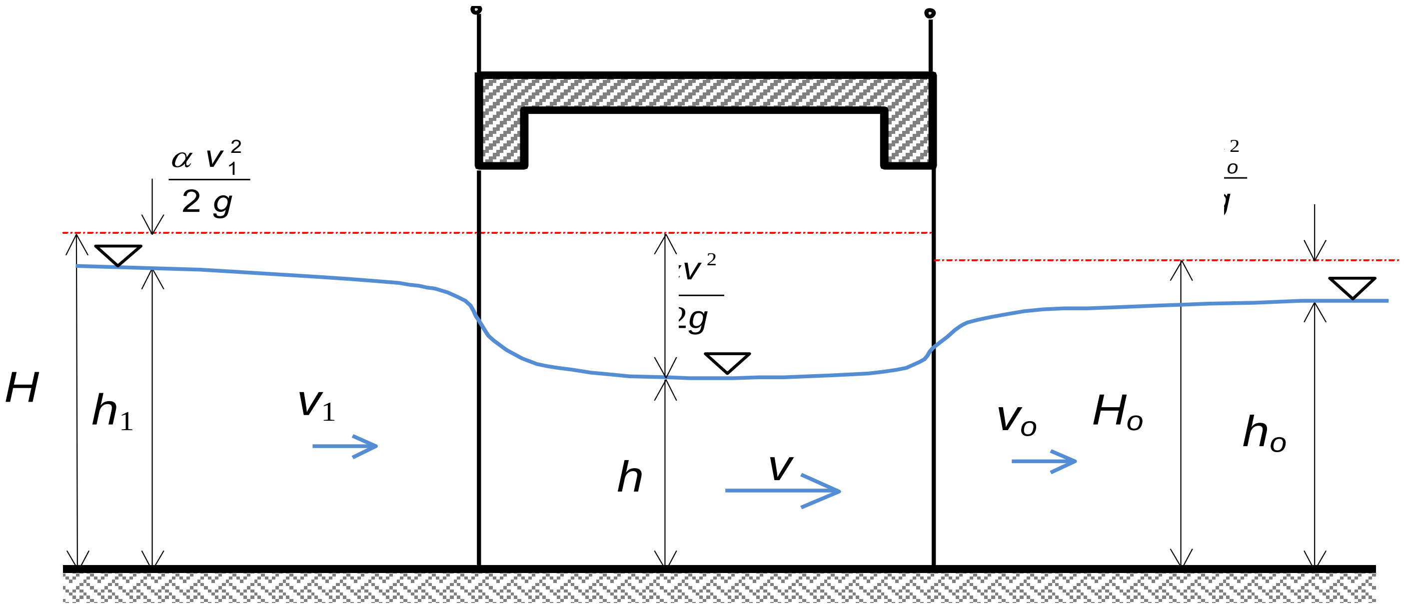
- unsubmerged,
- boxy shape not circular,
- 1 opening.

Maximum non-erodible (allowable) velocity

Original material excavated for canals	Mean velocity, for straight canals of small slope, after aging with flow depths less than 3 ft (0.9 m)					
	Clear water, no detritus		Water transporting colloidal silts		Water transporting noncolloidal silts, sands, gravels, or rock fragments	
	ft/s	m/s	ft/s	m/s	ft/s	m/s
Fine sand (noncolloidal)	1.5	0.46	2.5	0.76	1.5	0.46
Sandy loam (noncolloidal)	1.75	0.53	2.5	0.76	2.0	0.61
Silt loam (noncolloidal)	2.0	0.61	3.0	0.91	2.0	0,61
Alluvial silt (noncolloidal)	2.0	0.61	3.5	1.07	2.0	0.61
Ordinary firm loam	2.5	0.76	3.5	1.07	2.25	0.69
Volcanic ash	2.5	0.76	3.5	1.07	2.0	0.61
Stiff clay (very colloidal)	3.75	1.14	5.0	1.52	3.0	0.91
Alluvial silt (colloidal)	3.75	1.14	5.0	1.52	3.0	0.91
Shales and hardpans	6.0	1.83	6.0	1.83	5.0	1.52
Fine gravel	2.5	0.76	5.0	1.52	3.75	1.14
Graded, loam to cobbles (when oncolloidal)	3.75	1.14	5.0	1.52	5.0	1.52
Graded silt to cobbles (when colloidal)	4.0	1.22	5.5	1.68	5.0	1.52
Coarse gravel (noncolloidal)	4.0	1.22	6.0	1.83	6.5	1.98
Cobbles abd shingles	5.0	1.52	5.5	1.68	6.5	1.98

Source: Stream Restoration Design

Calculation schema for a bridge



- Continuity equaton:

$$Q = b\,h\,v \quad \rightarrow \quad b \geq \frac{Q}{\mu\,h\,v_d}$$

$-\ \mu(b) = 0.83 \div 0.99 \rightarrow$ iterative calculation

Calculation of the bridge opening in case of unerodible riverbed

Condition for water-table to be not elevated:

The largest channel conveyance is when: $h = h_k$

$$\frac{Q}{b} < q_{max} = v_k\, h\text{, where}\quad v_k = \sqrt{\frac{2}{3}\frac{H_o\, g}{\alpha}}$$

Calculation of the water depth under the bridge:

$$H_o = h_o + \frac{\alpha\, v_o^2}{2\,g} \rightarrow \quad h = H_o - \frac{\alpha\, v_d^{\,2}}{2\,g} \quad \rightarrow \quad b \geq \frac{Q}{\mu\left(H_o - \frac{\alpha\, v_d^2}{2\,g}\right)v_d}$$

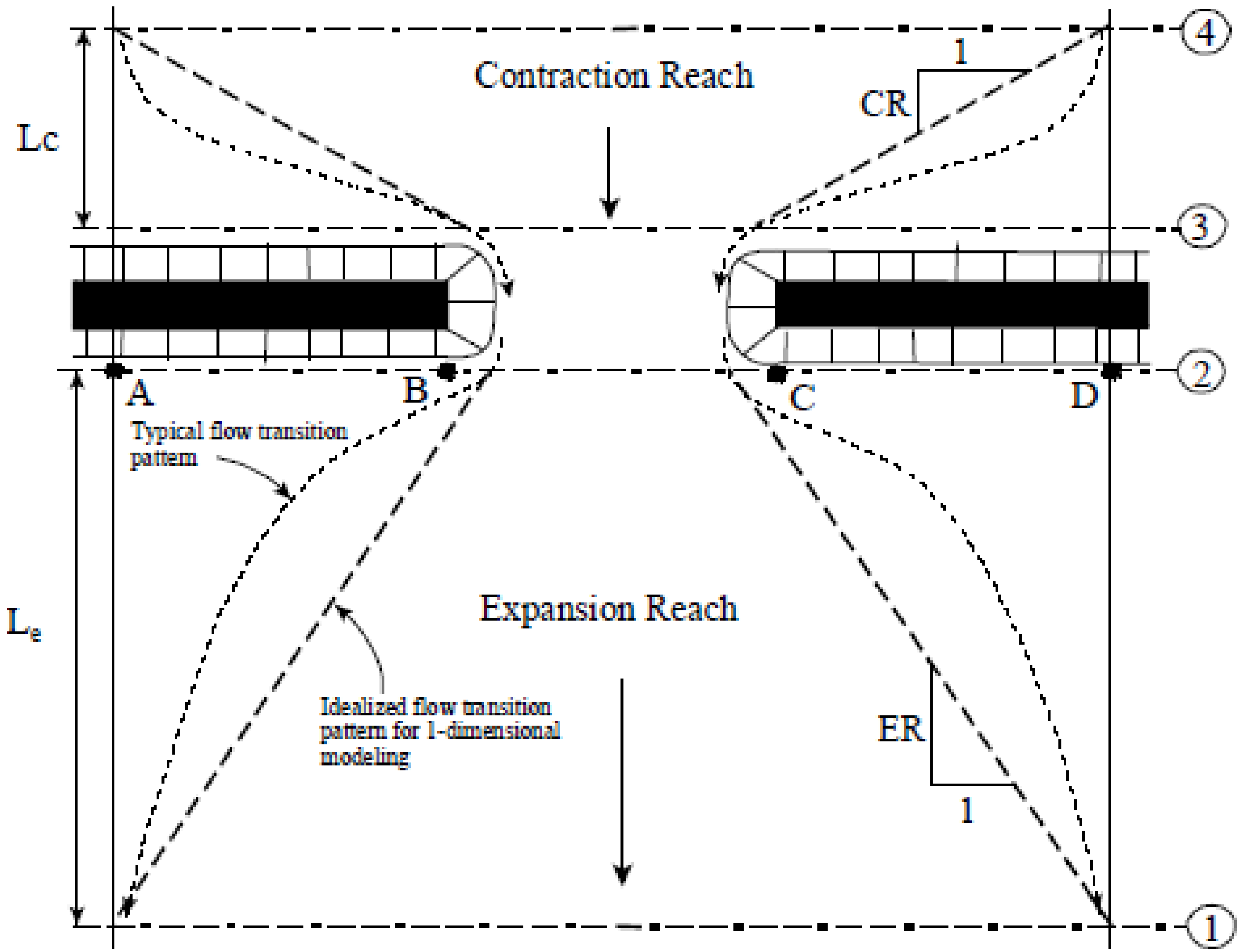
Calculation of the water depth just upstream of the bridge:

based on Bernoullie equation: $\rightarrow$ by iteration

$$h_1 = h + \frac{\alpha\left(v_d^2 - v_1^2\right)}{2\,g} + \zeta\,\frac{\alpha\, v_d^{\,2}}{2\,g}$$

ζ – resulting from flow contraction, abutments and piers of the bridge

Control cross-sections for flow field calculation



Source: Hec-Ras software User Manual

Calculation of the bridge opening in case of erodible riverbed

- Calculation of the erosion depth:
  by empirical formula:
  - ✓ expected increase of the depth (h_r)
  - ✓ scour at the piers
  - ✓ degree of erosion: $p = h_r / h_n$ − 1÷1.4
    (depends on the type of bridge foundation)
- Calculation of the bridge opening:
  based on sediment transport continuity
  $$b_r = \frac{b}{p^{\frac{3}{2}}}$$
- Calculation of elevated depth just upstream of the bridge:
  - ✓ water elevation without erosion:
    (the same formula as used previously)
  - ✓ depth of elevated water increased by scour:

$$h_r = h_{tail} + \left(h_{up} - h_{tail}\right)\left(\frac{A}{A_r}\right)^{\frac{8}{5}},\ A_r \cong p\,A$$ (in the case of whole opening scour)

Flow condition at the piers leading to a scour

Elevated watertable and acceleration of flow at the pillars
of the Grunwaldzki and railway bridge in Kraków
(increased velocity magnitude at the pier walls cause
increased bottom erosion in these places)

Calculation of the small bridge minimum opening

Continuity equation:

$$b = \frac{Q}{\mu\, h\, v_d}$$

- μ = 0.83÷0.94 used in the case of abutments,

Critical flow condition under the bridge:

- $v_d > v_k$ (critical velocity): $v = v_d \leftrightarrow v_k\left(H_o\right) = \sqrt{\frac{2\,g\,H_o}{3\,\alpha}}$

Calculation of the depth under the bridge:

$$h = h_k = \frac{2}{3} H = 2\,\frac{\alpha\, v_d^{\,2}}{2\,g} = \frac{\alpha\, v_d^{\,2}}{g} \quad \rightarrow \quad b \geq \frac{g\,Q}{\mu\,\alpha\, v_d^{\,3}}$$

Calculation of the elevated depth just upstream the bridge

By assuming a rectangular cross-section of the channel under the bridge and an opening width *b*, the velocity based on the flow rate can be calculated as:

hence:

$$\frac{\alpha v^2}{2g} = \frac{\alpha Q^2}{2g\ b^2\ h^2} \qquad v = \frac{Q}{b\ h}$$

After substitution into Bernoullie equation, a polynomial equation with unknown variable *h* is obtained:

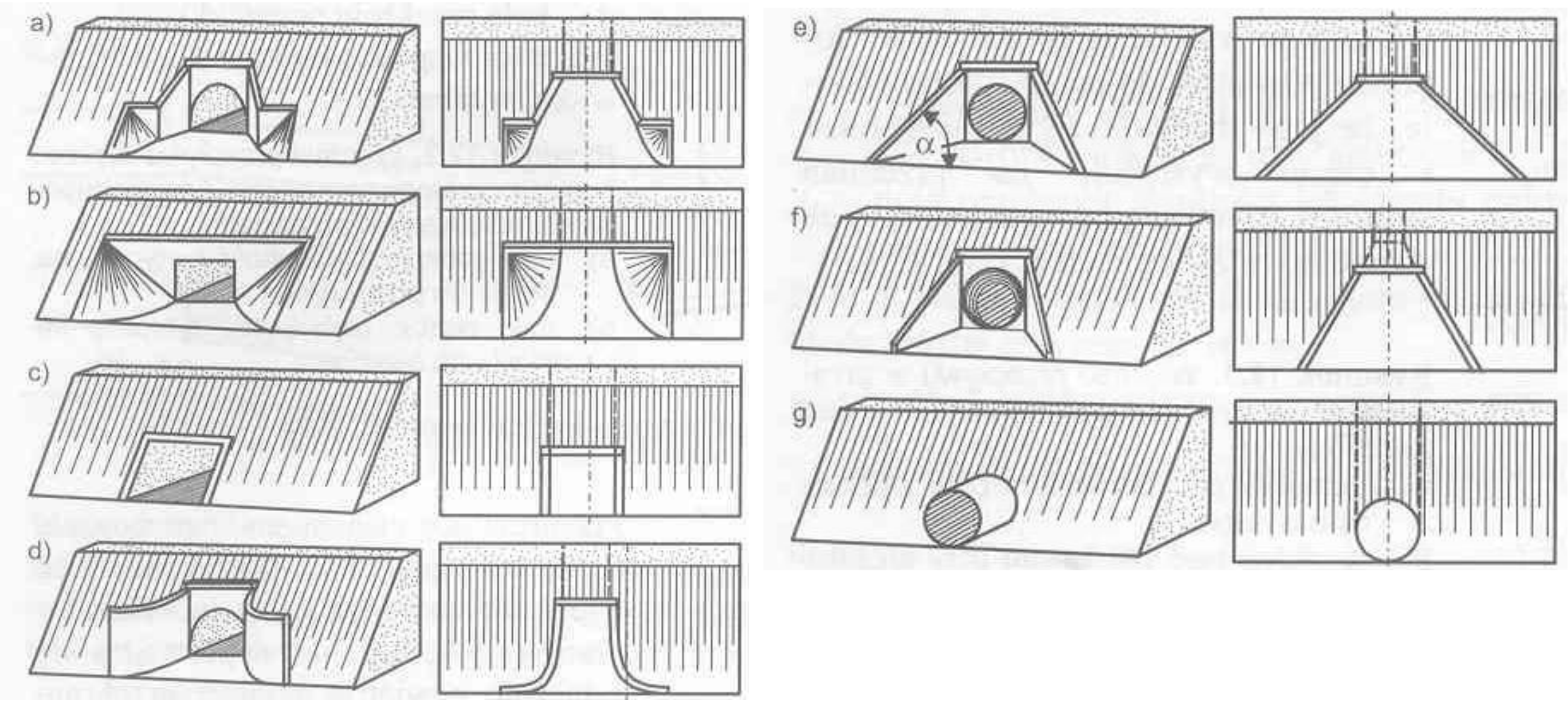
$$H = h_1 + \frac{\alpha}{2g}\frac{Q^2}{B^2 h_1^2} = 3\frac{\alpha v_d{}^2}{2g}$$

- The depth is one of the roots of the above equation:

$\cancel{h_1' < 0}$ $\qquad$ $\cancel{0 < h'' < h_k}$ $\qquad$ $h_1''' > h_k$

Calculation of culvert opening

Types of culvert entrance



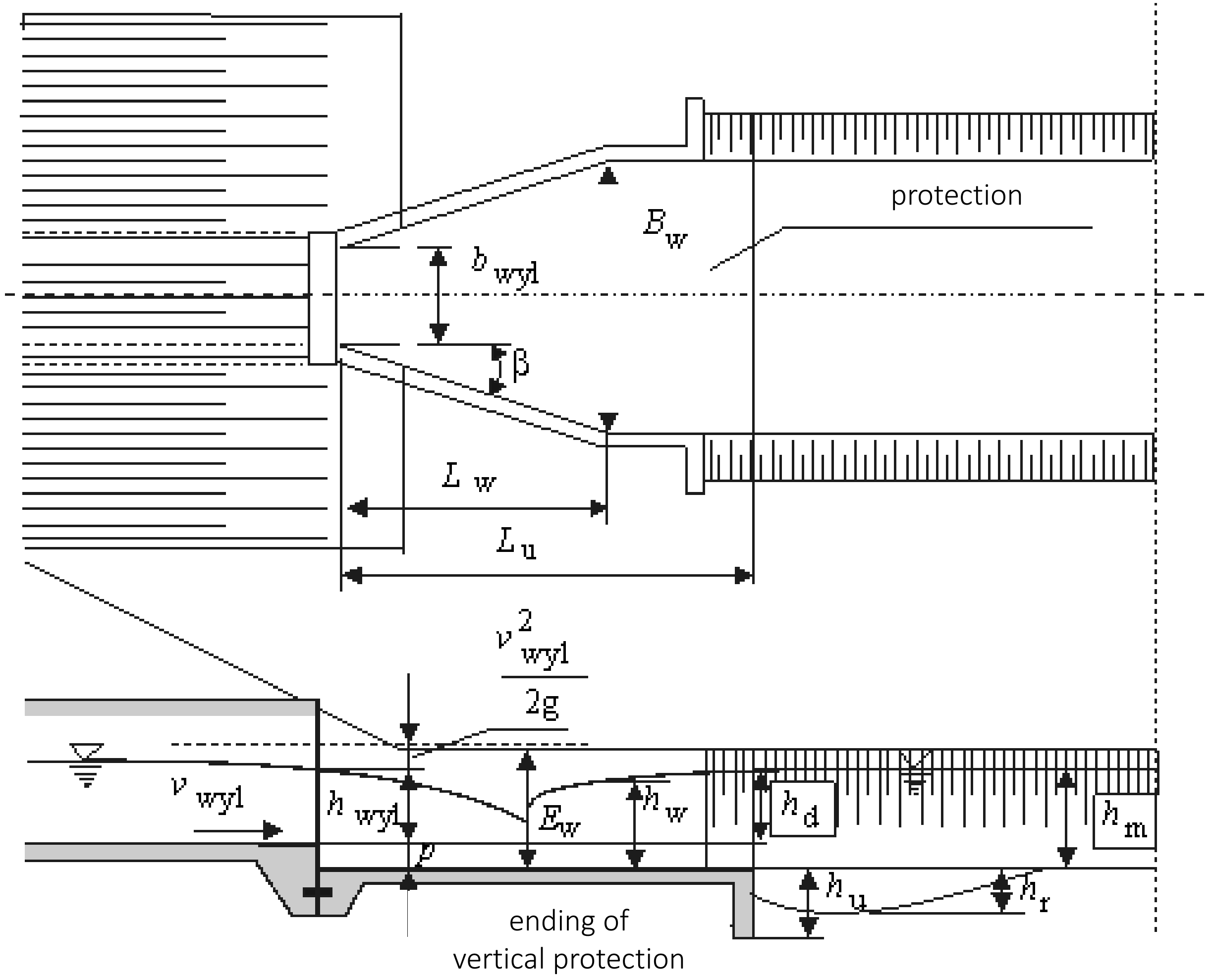
a – wings ended with cons,
b – perpendicular with cons,
c – inclined entrance,
d – with curved wings

e – with wings,
f – with wings and increased inlet,
g – with pipe inlet

Source: Kubrak, Kubrak 2010

Designing of a stilling basin for culvert

Protection of the basin floor



Regulation concerning the culverts conveyance designing

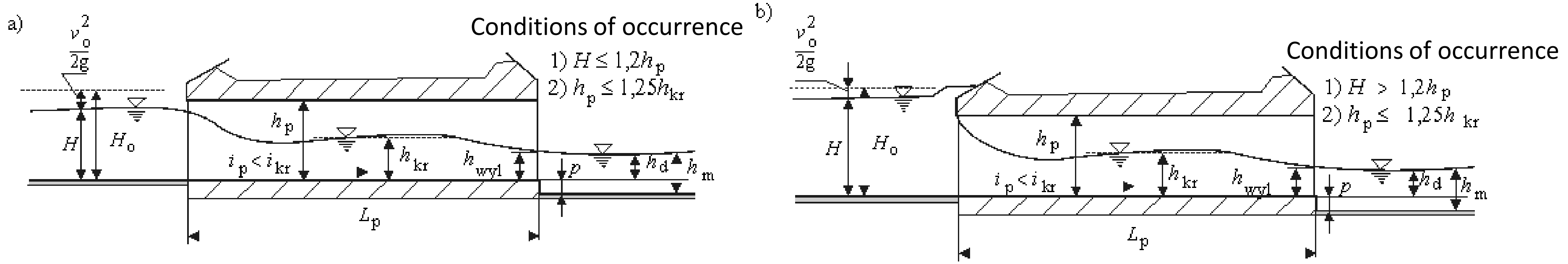
Culverts of rectangular and circular cross-sections should have a width opening:

1) not smaller than 1 m, in the case of roads of A and S types,
2) not smaller than 0.8 m, in the case of roads of GP, G and Z types.

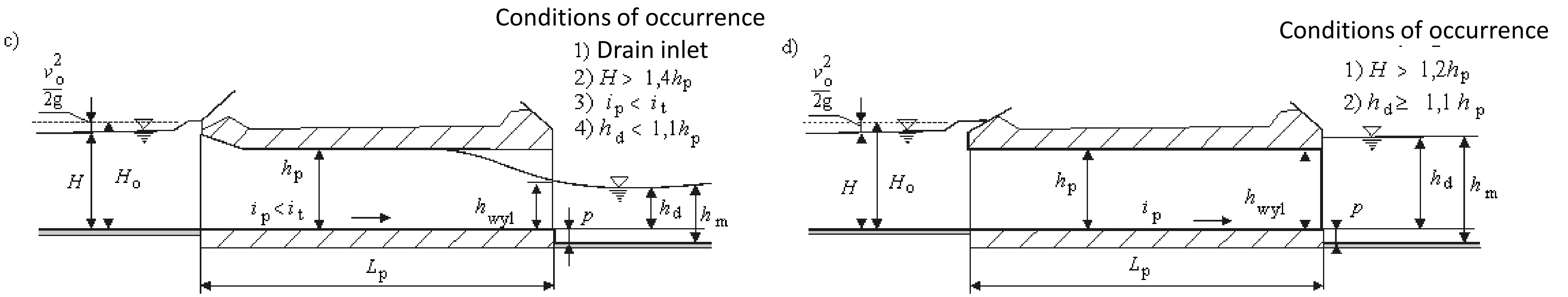
The height of culverts should be:

1) not less than 0.8 m if pipe is not longer than 20 m under road of types L and D,
2) not less than 1 m if pipe is not longer than 20 m under roads of remaining types,
3) not less than 1.2 m if pipe is longer than 20 m for all road types.

Hydraulic schemas of culvert operation



Free flow with unsubmerged entrance free flow with submerged entrance



Pressurized flow with unsubmerged outlet pressurized flow with submerged outlet

Formulas used for opening size calculation

Culvert with free surface flow:

- the same Formula as used for broad-crested weir:

$$Q = m\, b\, \sqrt{2\, g}\; H_o^{3/2}$$

Culvert with free surface flow and submerged inlet:

- as in the case of sluice gate outflow rate calculation:

$$Q = \mu\, A\, \sqrt{2\, g\; (H_o - \varepsilon\, h_p)}$$

Formulas used for opening size calculation

Pressurized culvert:

- as in the case of pipe flow approach calculation:
  - rectangular cross-section

$$Q = b\,h_p \sqrt{\frac{2\,g\,\Delta H}{\zeta + \lambda \dfrac{l}{4\,R} + 1}} \qquad \lambda = \frac{8\,g\,n^2}{\sqrt[3]{R}}$$

  - circular c.s.

$$Q = \frac{\pi\,{d_k}^2}{4} \sqrt{\frac{2\,g\,\Delta H}{\zeta + \lambda \dfrac{l}{d_k} + 1}} \qquad \lambda = \frac{8\,\sqrt[3]{4}\,g\,n^2}{\sqrt[3]{d_p}}$$

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